A SIMPLE LARGE SIGNAL MODEL FOR ISOLATED DC-DC CONVERTERS

Wilson Eberle, Yan-Fei Liu and P.C. Sen
Queen's Power Group, www.queenspowergroup.com
Queen's University, Department of Electrical and Computer Engineering, Kingston, Ontario, Canada, K7L 3N6
wilsoneberle@wilsoneberle.com, yanfei.liu@queensu.ca and senp@post.queensu.ca

Abstract

In this paper, a large signal model for isolated DC-DC converters is proposed. The model is applicable to both current and voltage mode control. The model has the principle advantage that it is simple to derive - it takes the same form as the switching converter that it is derived from. In the model, the MOSFET switches are replaced by dependent current and voltage sources equal to the average current through the switch, or average voltage across the switch. The active switch and its corresponding synchronous rectifier are replaced by dependent average current sources. In multiple switch topologies, the additional switches are replaced by dependent average voltage sources. Implementation of the model is very simple since no mathematical derivations are required. The only change to the circuit is the replacement of the switches by their dependent sources. The model is simple to implement in circuit simulation software packages such as SPICE. Once implemented, small signal and large signal behaviour can be obtained through simulation. The model is verified experimentally for the small signal and large signal cases using a prototype of the asymmetrical half-bridge topology operating at 48V input, and 5V at 6A load and a 400kHz switching frequency. Good agreement is obtained between the simulation and experimental results validating the model.

Keywords: *Modeling, current mode control, converters.*

1. Introduction

By their very nature, switching power supplies are time-variant and non-linear. As a result, conventional linear control techniques cannot be directly applied to analyze converter dynamics. However, to design an appropriate feedback system that can assure stability and good dynamic performance, we need a dynamic model of the switching converter. The dynamic model should model the dominant low frequency behaviour of the system, but should neglect behaviour at and beyond the switching frequency.

Many small signal models have been developed for switching converters, including [1]-[5], which model current mode converters. However, these models can only predict small signal stability in the vicinity of their operating point, but converters can become unstable when they experience a large perturbation, such as large transient load changes which are common for present day power systems. Therefore, a large signal model is essential to study the dynamic characteristics of all switching converters, including those under voltage mode control and current mode control. Furthermore, the existing modeling methods, including the three terminal PWM switch in [6] are complicated to derive and the final circuit model does not resemble the converter that it is derived from, so it is difficult to easily determine by inspection if the model is correct.

In this paper, a very simple large signal modeling method for isolated DC-DC converters under current mode control is presented based on the averaged circuit model presented in [7]. However, the model in [7] is only applicable to simple, single-switch, non-isolated topologies such as the buck converter. Issues, such as how to model multiple switch topologies, synchronous rectifiers and how to model a transformer are not discussed.

The proposed model uses the state-space averaging technique and takes the form of the averaged circuit model that has the same topology as that of the switching converter. The switches are modeled by dependant sources and all other non-switching, passive devices are not modeled i.e. they remain as their circuit elements. Once the large signal model is established, the large signal characteristics can be analyzed in the time domain using the differential equations, or by using a circuit simulation software package, eg. SPICE. The small signal characteristics can be obtained by using the AC analysis tools in SPICE, or the small signal transfer functions can easily be derived from the large signal model. There are many advantages to the proposed model:

Simple: The averaged circuit model has the same topology as the switching converter. In addition, the dependent sources used to model the switches are easily derived from the switching voltage, or current waveform.

General: The model can be applied to all isolated and non-isolated DC-to-DC converters under current, or voltage mode control.

Powerful: The effects of slope compensation, parasitics and filtering can be easily included.

Unified: The small signal transfer functions and the steady-state input-output relations can be derived from the large signal averaged circuit model.

2. Deriving the Averaged Model for Isolated DC-DC Converters

2.1. Voltage Mode Control Derivation

The output voltage of all PWM DC-DC converters is controlled by the duty cycle of the active switch. The duty cycle is the duty ratio, *d*, which is the control variable for voltage mode converters. For current mode converters, the duty cycle is controlled by the peak current, which is the control variable. Due to the simplicity of voltage mode control, it is convenient to first derive the averaged circuit model for isolated DC-DC converters under voltage mode control.

Utilizing the state space averaging method [8] and basic circuit theory, for every set of state space differential equations, a circuit topology can be found that has the same state space differential equations. This argument provides another possibility to express the averaged model as the averaged circuit model in which every circuit variable is the averaged value of the corresponding instantaneous variable.

The AHB, Fig. 1(a), is used to illustrate this concept. The waveforms to be averaged for the four AHB switches are given in Fig. 1(b). The following assumptions have been made: 1) all ripple components can be neglected, 2) the MOSFETS are ideal with no conduction voltage drops, no switching loss and no switching time, and 3) there is no dead-time between MOSFET on-off state transitions.

It is proposed that all switches that conduct during T_{ON} (Q_1 and Q_3) are modeled by controlled current sources equal to the average current through the switch during one switching period and all switches that conduct during T_{OFF} (Q_2 and Q_4) are modeled by controlled voltage sources equal to the average voltage across the switch during one switching period. Therefore, the averaged circuit model is obtained as shown in Fig. 1(c). Care should be taken when deriving the averaged sources. The averaged sources should be derived from waveforms which have not already been averaged. This is evident in the example above where i_{Q1} and v_{Q3} must be derived from their reflected original source waveforms e.g. i_F and i_M for i_{Q1} and v_{IN} and v_{CB} for v_{Q3}.

2.2. Current Mode Control Derivation

Current mode control is widely used because of its advantages over voltage mode control, such as fast response, improved damping and over current protection. Unfortunately, because there are two feedback loops, the analysis of the dynamic characteristics of current mode converters is difficult. Fortunately, the current loop can be modeled using the approach presented below.

Fig. 2 gives the detailed relationship between the control signal, i_C and the Q_1 switch current, $i_{Q1}(t)$, which is used to derive the duty ratio. In Fig. 1(a), $i_{Q1}(t)$ denotes the instantaneous switch current and i_{Q1} denotes the state-space averaged switch current during the interval dTs. An optional artificial stabilizing ramp has been included with slope M_a. The state-spaced averaged switch current passes through the midpoint of the actual switch current, so i_{Q1} is derived from the geometry (1).

$$i_{Q1} = i_C - 0.5 m_1 d T_S - d M_a T_S \tag{1}$$

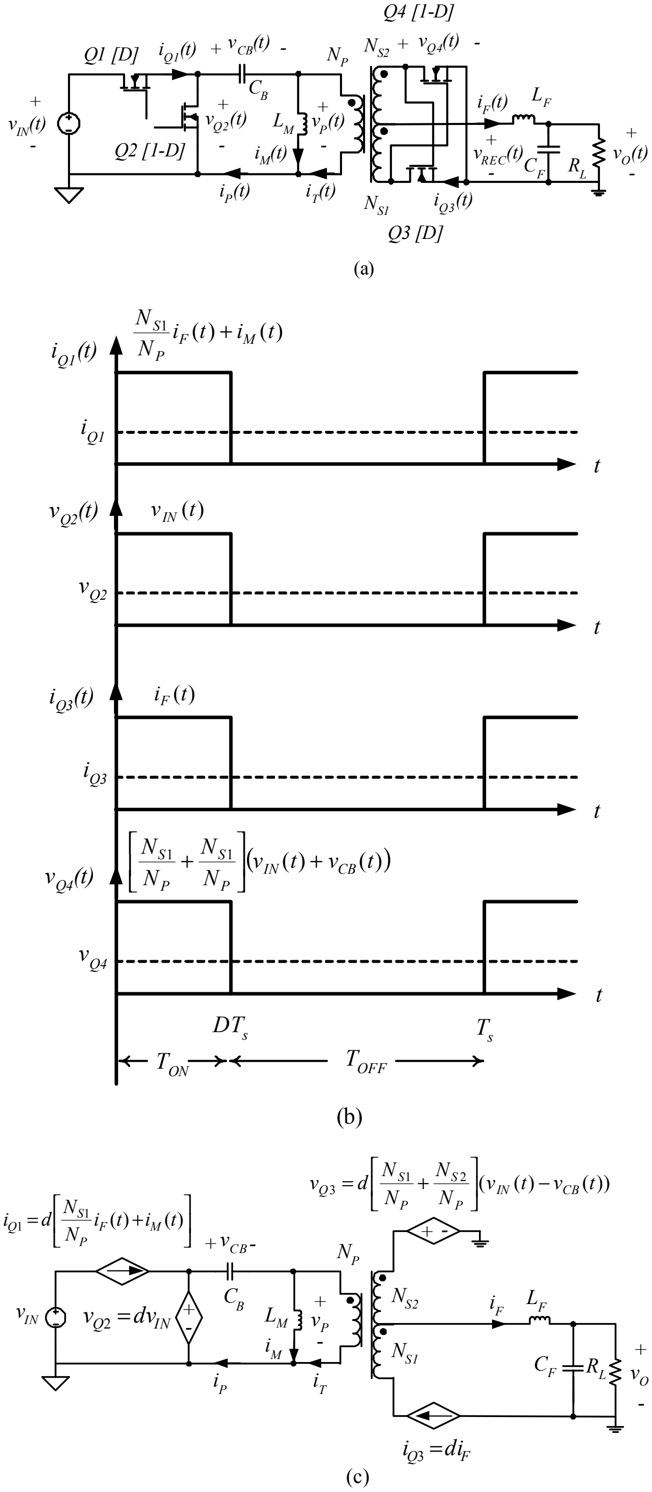


Fig. 1 Derivation of the averaged circuit model for the AHB (a) AHB, (b) AHB switching waveforms, (c) AHB averaged circuit model

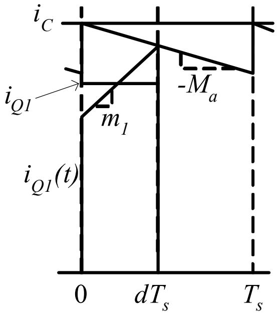


Fig. 2 Detailed waveforms between switch current, i_{Q1} and control signal, i_C for current mode control

For the AHB, since the primary current, i_P is equal to i_{Q1} during T_{ON}, we can replace i_{Q1} in (1) with i_P. In current mode control, the duty ratio, d is not the direct control variable, but it can be expressed by (2), which holds true for all switching converters. For the AHB converter, it is expressed as (3) where m_1 is the primary current slope and no artificial stabilizing ramp has been included (M_a=0). For the AHB, m_1 can be expressed as (4). The large signal current mode control model for the AHB is then obtained by substituting equations (3) and (4) for d into the model of Fig. 1(c). It should be noted that the averaged sources are not independent, but are controlled by other circuit variables, such as i_C, i_F, v_{IN} and so forth. Furthermore, since all the circuit variables are the state-spaced averaged value and because no small signal assumption is imposed during the derivation, the model is a large signal model for the current mode controlled AHB.

$$d = \frac{i_C - i_{Q1}}{0.5 m_1 T_S + M_a T_S} \tag{2}$$

$$d = \frac{i_C - \left(\frac{N_{S1}}{N_P} i_F + i_M \right)}{0.5 m_1 T_s} \tag{3}$$

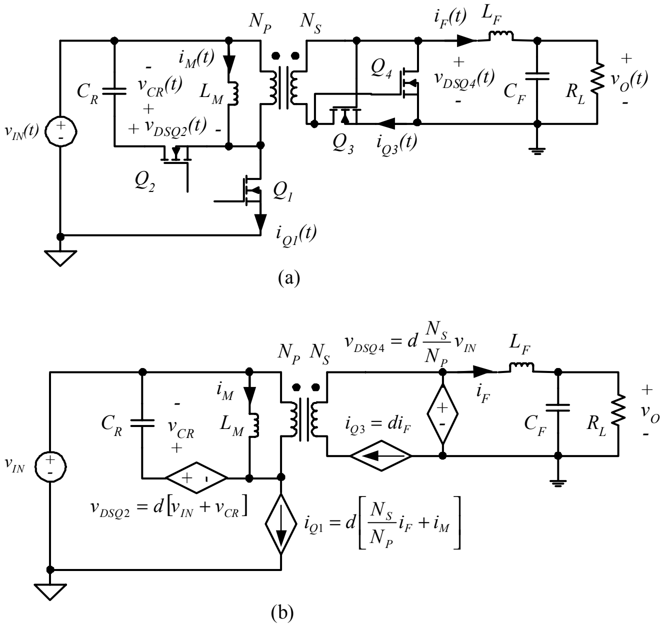
$$m_1 = \frac{(v_{IN} - v_{CB})}{L_M} + \frac{N_{S1}}{N_P} \frac{\left[\frac{N_{S1}}{N_P} (v_{IN} - v_{CB}) - v_o \right]}{L_F} \tag{4}$$

2.3. The Small Signal Model

The small-signal model can be derived by perturbing and linearizing the averaged circuit dependent sources. However, a significant advantage of the proposed model is that the small signal characteristics (bode diagrams) can easily be obtained using the large signal model and the AC analysis tools with SPICE, so deriving the transfer functions has diminished value, because the simulation software can quickly produce the bode diagrams.

3. Other Isolated DC-to-DC Converter Topologies

The averaged circuit model can be derived for all isolated DC-DC converter topologies. The averaged circuit model for the active clamp forward (ACF) is shown below in Fig. 3. It should be noted that the ACF model closely resembles the topology. In addition, the dependent sources in the model are simple to derive from the converter switch waveforms.



(a)

(b)

Fig. 3 Active Clamp Forward Converter, (a) Topology, (b) ACF averaged circuit model

4. Experimental Results

A prototype of the AHB has been built on a printed circuit board to verify the model. The input voltage was 48V. The output voltage was 5V. The full load current was 6A and the switching frequency was 400kHz. The voltage loop was closed using compensation to maximize the bandwidth while maintaining a phase margin of at least 45 degrees and a gain margin of at least 10dB. The model was simulated using SPICE with additional damping resistances added to represent the real circuit parasitics.

The small signal loop response of the model is given in Fig. 4. The experimental verification is given in Fig. 5. The large signal model was verified for a step change in load current from 5A to 6A. The model output voltage transient is shown in Fig. 6 and the experimental verification is shown in Fig. 7. In both the small and large signal cases, the model results match the experimental results very closely.

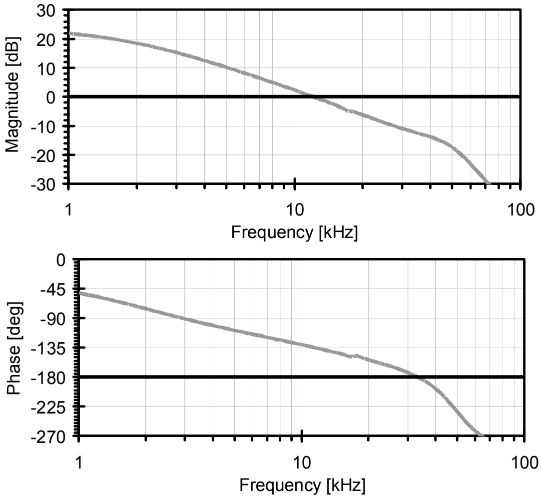


Fig. 4 Model loop response; top curve: magnitude; bottom curve: phase; 5V/6A output at 48V input

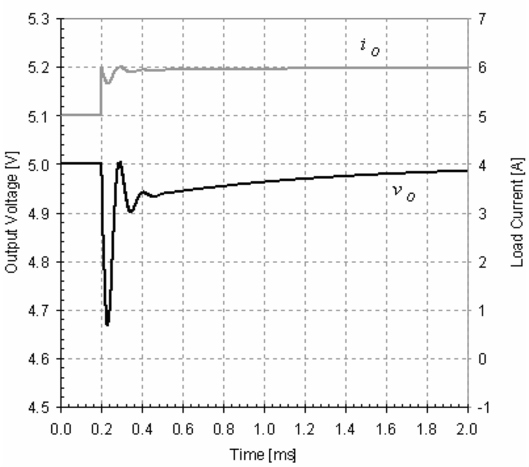


Fig. 6 Model output voltage transient for a 1A step in load current from 5A to 6A; top curve: load current; bottom curve: output voltage, 100mV/div; 5V output

5. Conclusions

An averaged circuit model for isolated DC-DC converters has been presented. The model is applicable to both current and voltage mode control. The model has the principle advantage that it is very simple to derive - it takes the same form as the switching converter that it is derived from, so the model circuit appears identical to the switching converter with the exception that the switches have been replaced by averaged dependent sources. Furthermore, the model is a large signal model. The small signal characteristics can be derived from the model, or they can be obtained using a simulation software package. It has been demonstrated experimentally that the small signal and large signal behaviour of the model match very closely to the switching converter.

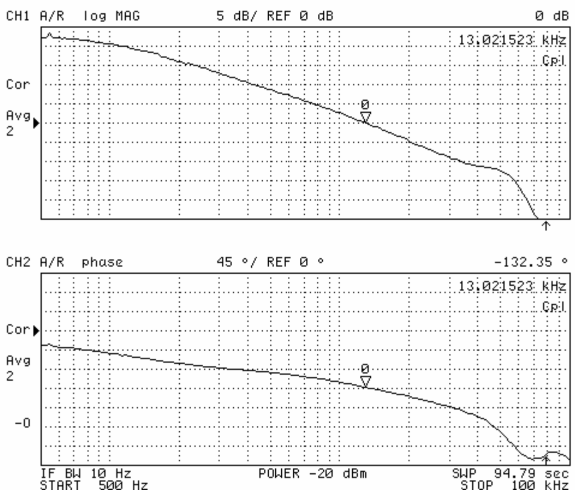


Fig. 5 Experimental loop response; top curve: magnitude; bottom curve: phase; 5V/6A output at 48V input

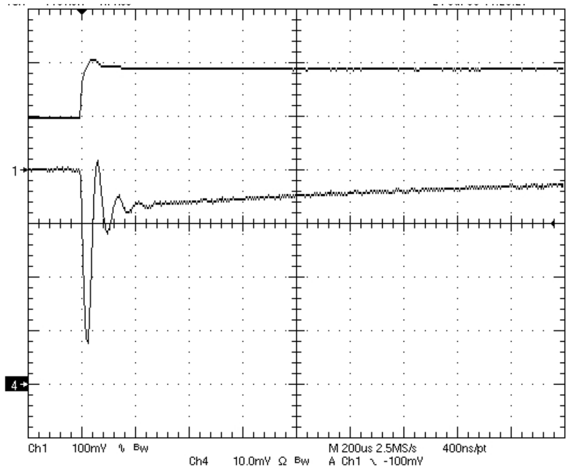


Fig. 7 Experimental output voltage transient for a 1A step in load current from 5A to 6A; top curve: load current; bottom curve: output voltage, 100mV/div; 5V output

References

[1] S.P. Hsu, A. Brown, L. Rensink and R.D. Middlebrook, "Modelling and Analysis of Switching DC-to-DC Converters in Constant-Frequency Current-Programmed Mode," IEEE Power Electronics Specialists Conference (PESC), 1979, pp. 284-301.

[2] R.D. Middlebrook, "Modeling Current-Programmed Buck and Boost Regulators," IEEE Transactions on Power Electronics, Vol. 4, January 1989, pp. 36-52.

[3] R. Tymerski and D. Li, "State Space Models for Current Programmed Pulsewidth Modulated Converters," IEEE Power Electronics Specialists Conference (PESC), 1992, pp. 337-344.

[4] R. W. Erickson and D. Maksimovic, "Fundamentals of Power Electronics, Second Edition," Kluwer Academic Publishers, 2001.

[5] V. Vorperian, "Analysis of Current-Mode Controlled PWM Converters using the Model of the Current-Controlled PWM Switch," Power Conversion Conference (PCIM), 1990, pp. 183-195.

[6] V. Vorperian, "Fast Analytical Techniques for Electrical and Electronic Circuits," Cambridge, 2002.

[7] Y.F. Liu and P.C. Sen, "A General Unified Large Signal Model for Current Programmed DC-to-DC Converters," IEEE Transactions on Power Electronics, Vol. 9, No. 4, July 1994, pp. 414-424.

[8] R.D. Middlebrook and S. Cuk, "A General Unified Approach to Modeling Switching Converter Power Stages," IEEE Power Electronics Specialists Conference (PESC), 1976, pp. 18-34.