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FUNCTION CONTROL AND ITS APPLICATION IN DC-DC SWITCHING REGULATORS

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ABSTRACT

The control strategy of a dc-dc switching regulator is studied in view of the mathematical relation between the inputs and the output of a control circuit, and a function control is suggested, by which the regulator is inherently stable,the effects of the disturbances from the line voltage and the load resistance can be eliminated and greatly reduced,respectively. The control circuit is simple and flexible. As an example, an experimental function control Buck regulator is breadboarded to verify the analysis.

I. INTRODUCTION

A good control strategy can greatly improve the performance of a dc-dc switching regulator. The duty ratio control and the current programming control are most commonly applied . The block diagram of the duty ratio control circuit is simple , but a network must be designed to compensate the defects of right-half-plane zeroes of some topologies of power stage and it is less capable against the perturbations of the line voltage and the load[1]. The current programming control is superior to the duty ratio in its faster response, automatic current-limitting and current-sharing [2,3]. Because of the inherent instability and subhomonic oscillation of the current programming control [4,5], a compensating network must be added and the parameters carefully selected to keep the regulator stable. The existance of the inner loop and the outer loop makes it very inconvenient and time consuming to analyse and design the current programming switching regulator [5,6,8]. Moreover, the control circuit of the above mentioned two controls is not flexible and is difficult to be adjusted.

It is therefore beneficial to look for a new control strategy that can keep the regulator inherently stable, eliminate or depress to a great extent the perturbations of the line voltage and the load. And at the same time its control circuit is flexible and easy to implement. It is indeed the motive and the objective of the paper.

In section II, the common point of various controls are analysed and the function control is proposed. Its application to the four basic switching regulators (Buck, Boost, Buck-Boost and Cuk) is illustrated in section III. Section IV gives a simple function control circuit. A function control Buck regulator is breadboarded in section V to verify the analysis. Section VI is the conclusion.

II. FUCTION CONTROL PRINCIPLE

A dc-dc switching regulator consists of two parts i.e. the power stage and its control circuit. They connect each other by the feedbacks and/or feedforwards, x , and the duty ratio, d , as illustrated in Fig. 1 ,where $x = (x_1, x_2, \dots, x_n)$ represents a combination of circuit variables in the power stage, r is load resistance, v_g the line voltage, v_0 the output voltage, and they all are time dependent.

In one hand, according to the chosen feedbacks and/or the feedforwards , x , and the relevant control strategy,a control circuit is expected to produce a series of pulse with duty ratio, d, in order that the output voltage can resume to its original value as much as possible when perturbed. Thus although the operating principles of various control strategy are diferent from one another, a common point can be seen that the output of the control circuit can be expressed as the function of its inputs and the time , mathematically:

$$d = g(x,t) \qquad (1)$$

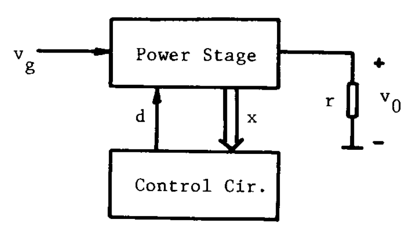

Fig. 1. General block diagram of a dc-dc switching regulator

From eq. (1) , the steady-state and the small signal dynamic relations of the control circuit can be deduced by the perturbation technique as follows:

$$D = g(X) \tag{2}$$

$$\hat{d} = g_1\hat{x}_1 + g_2\hat{x}_2 + \dots + g_n\hat{x}_n \tag{3}$$

where $X = (X_1, X_2, \dots, X_n)$ and $\hat{x} = (\hat{x}_1, \hat{x}_2, \dots, \hat{x}_n)$ are the steady-state value and the small signal perturbation of x, respectively; $x = X + \hat{x}$; $g_1, g_2, \dots, g_n$ are coefficients; and D is the steady-state value of d, and $\hat{d}$ its small signal perturbation, $d = D + \hat{d}$.

On the other hand, the steady-state and the small signal dynamic relations of a power stage can be expressed as follows:

$$V_0 = G_v U \tag{4}$$

$$\hat{v}_0 = \frac{G_1(s)\hat{u} + G_2(s)\hat{d} + G_3(s)\hat{r}}{H_p(s)} \tag{5}$$

where G_v is the steady-state gain. U is the steady-state value of u, $\hat{u}$ is its small signal perturbation, while u is a circuit variable of the power stage and may be the line voltage v_g, the inductor current i_L or the capacitor voltage v_C. $\hat{r}$ is the small signal perturbation of r.

Obviously, the output of the control circuit, d, is the same quantity as the control input of the power stage and the inputs of the control circuit are circuit variables of the power stage.Substitute eq. (2) to eq. (4), eq. (3) to eq. (5), the closed loop steady-state and small signal dynamic relations of a switching regulator are as follows:

$$V_0 = n_1V_r + n_2V_g + n_3R \tag{6}$$

$$\hat{v}_0 = \frac{C_1(s)\hat{v}_g + C_2(s)\hat{r}}{H(s)} \tag{7}$$

where R is the steady-state value of r, n_1, n_2, n_3 are coefficients and V_r is the reference voltage.

Therefore the ultimate objective of control strategy study is to choose the proper inputs, x , and the function, g , so that the coefficients in eqs. (6) (7) meet the following demands:

$$n_1 = \text{const.},\ n_2 = 0,\ n_3 = 0 \tag{8}$$

$$C_1(s) = 0,\ C_2(s) = 0, \text{ and the roots of } H(s) = 0 \text{ on the left half plane} \tag{9}$$

From the above analysis and taking it into account that the function, g , should be simple, and the output of a control circuit is dimensionless while its inputs are dimensional, a function relation between the inputs and the output of a control circuit may be formulated as follows:

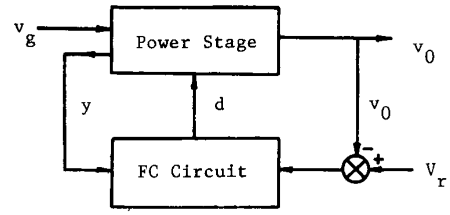

Fig. 2. General block diagram of a FC switching regulator

$$d = \frac{a_1y + a_2(V_r - v_0)}{b_1y + b_2(V_r - v_0)} \tag{10}$$

where $(V_r - v_0)$ denotes the negative feedback, coefficients a_1, a_2, b_1, b_2 and feedforward y are to be chosen according to given critera. It is called the function control for the reason that a simple function relation between the inputs and the output of a control circuit is used here to generate the control pulse of the power stage. Fig. 2 gives the general block diagram of function control (FC) switching regulator.

From eq. (10) we can get the steady-state and small signal dynamic relations of the function control circuit:

$$D = \frac{a_1Y + a_2(V_r - V_0)}{b_1Y + b_2(V_r - V_0)} \tag{11}$$

$$\hat{d} = g_1\hat{y} + g_2\hat{v}_0 \tag{12}$$

where Y is the steady-state value of feedforward y and $\hat{y}$ its small signal perturbation, $y = Y + \hat{y}$, and g_1, g_2 are as follows:

$$g_1 = \frac{(a_1b_2 - a_2b_1)(V_r - V_0)}{[b_1Y + b_2(V_r - V_0)]^2} \tag{13}$$

$$g_2 = \frac{(a_1b_2 - a_2b_1)Y}{[b_1Y + b_2(V_r - V_0)]^2} \tag{14}$$

The critera to determine the feedforward y are: (a) to choose a proper u so that the expressions $G_1(s)$ and $G_2(s)$ in eq. (5) are frequency-independent and (b) to take the u as feedforward y. The feedforward thus selected can make the dynamic behaviour of the function control switching regulator

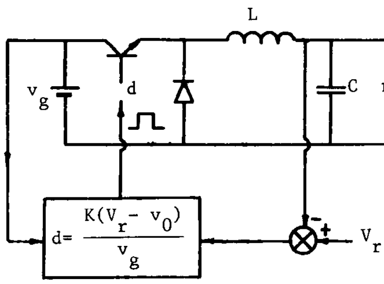

Fig. 3. FC Buck regulator

good. The coefficients a_1, a_2, b_1, b_2 are determined by eqs. (8) (9).

III. FUNCTION CONTROL SWITCHING REGULATOR

The Buck regulator is taken as an example. The steady-state and small signal dynamic relations of the power stage are:

$$V_0 = DV_g \tag{15}$$

$$\hat{v}_0 = \frac{D\hat{v}_g + V_g\hat{d} + (sLV_0/R^2)\hat{r}}{LCs^2 + Ls/R + 1} \tag{16}$$

From the analysis in section II, due to the $G_1(s) = D$ and $G_2(s) = V_g$, which are both frequency-independent, the line voltage v_g is taken as the feedforward y, as shown in Fig. 3, and function relation of the control circuit may be formulated as:

$$d = \frac{a_1 v_g + a_2(V_r - v_0)}{b_1 v_g + b_2(V_r - v_0)} \tag{17}$$

Its steady-state and small signal dynamic relations are as follows:

$$D = \frac{a_1 V_g + a_2(V_r - V_0)}{b_1 V_g + b_2(V_r - V_0)} \tag{18}$$

$$\hat{d} = g_1\hat{v}_g + g_2\hat{v}_0 \tag{19}$$

where $g_1 = (a_1 b_2 - a_2 b_1)(V_r - V_0)/[b_1 V_g + b_2(V_r - V_0)]^2$, $g_2 = (a_1 b_2 - a_2 b_1)V_g/[b_1 V_g + b_2(V_r - V_0)]^2$.

Substituting eq. (18) to eq. (15), eq. (19) to eq. (16), and choosing

$$a_1 = 0 \ , \ b_2 = 0 \tag{20}$$

and from eq. (17):

$$d = \frac{K(V_r - v_0)}{v_g} \tag{21}$$

where $K = a_2/b_1$, we can get the closed loop steady-state output voltage and the small signal transfer function of Buck regulator:

$$V_0 = \frac{K}{1 + K} V_r \tag{22}$$

$$\hat{v}_0 = \frac{0\cdot\hat{v}_g + (sL_d V_0/R^2)\hat{r}}{L_d C s^2 + L_d s/R + 1} \tag{23}$$

where $L_d = L/(1 + K)$. It is shown that by the function control, the steady-state output voltage of Buck regulator is proportional to the reference V_r. The effect of the line voltage can be completely eliminated and the change of the output voltage caused by the variation of the load resistance can be greatly reduced by choosing the proper value of K and the proper circuit parameters.

Analogously, taking the inductor current i_L in Boost convertor (Fig. 4) and Buck-Boost convertor (Fig. 5), but the capacitor voltage v_{C1} in Cuk convertor as the feedforward, y , we can obtain the similar results, as listed in Table 1 .

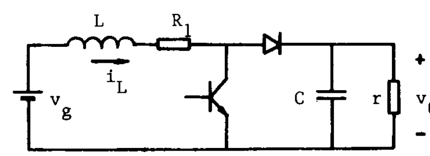

Fig. 4. Boost convertor $y=i_L$

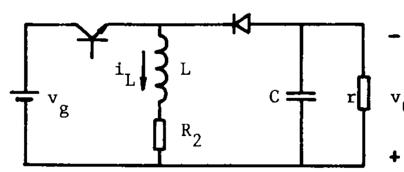

Fig. 5. Buck-Boost convertor $y=i_L$

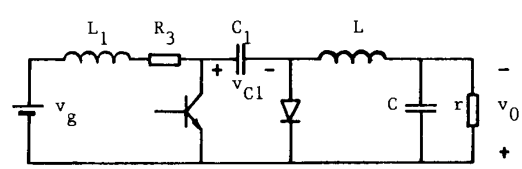

Fig. 6. Cuk convertor $y=v_{C1}$

Table 1. Function relations and closed loop characteristics of basic switching regulators

	y	d	V_0	$\hat{v}_0/\hat{v}_g$	$\hat{v}_0/\hat{r}$
Buck	v_g	$\frac{K(V_r-v_0)}{v_g}$	$\frac{K}{1+K}V_r$	$\frac{0}{H_{p1}(s)}$	$\frac{sL_dV_0}{R^2H_{p1}(s)}$
Boost	i_L	$\frac{i_L-K(V_r-v_0)}{i_L}$	$\frac{KR}{1+KR}V_r$	$\frac{0}{H_{p2}(s)}$	$\frac{sL_{e1}+R_{e1}}{R^2H_{p2}(s)}V_0$
Buck-Boost	i_L	$\frac{i_L-K(V_r-v_0)}{i_L}$	$\frac{KR}{1+KR}V_r$	$\frac{0}{H_{p3}(s)}$	$\frac{sL_{e2}+R_{e2}}{R^2H_{p3}(s)}V_0$
Cuk	v_{C1}	$\frac{K(V_r-v_0)}{v_{C1}}$	$\frac{K}{1+K}V_r$	$\frac{0}{H_{p4}(s)}$	$\frac{sL_dH_Z(s)}{R^2H_{p4}(s)}V_0$

In the Table 1 , $K = a_2/b_1$, $L_d= L/(1 + K)$, $L_{e1}= L_{e2}= L/(1 - D)^2$, $R_{e1}= R_1/(1 - D)^2$, $R_{e2}= R_2/(1 - D)^2$, $L_e= D^2L_1/(1 - D)^2$, $C_e= C_1/D^2$, $H_{p1}(s) = L_dCs^2+ L_ds/R + 1$, $H_{p2}(s) = (sL_{e1}+ R_{e1})(sC + 1/R + K)$, $H_{p3}(s) = (sL_{e2}+ R_{e2})(sC + 1/R + K)$, $H_{p4}(s) = H_Z(s)(L_dCs^2+ L_ds/R + 1)$, $H_Z(s) = L_eC_es^2+ R_3C_1s + 1$.

It can be concluded from Table 1 that a function control switching regulator possesses the following features:

1° the bad effects caused by the RHP zeroes can be dispelled;

2° the closed loop system is inherently stable;

3° the closed loop output voltage is independent of the line voltage;

4° the defects of load resistance can be limited to a given small value.

Another unique excellent characteristic is that an engineer can design a function control switching regulator very conveniently due to the fact that the closed loop small signal transfer function can be got easily.

Studying the $H_{p1}(s)$ in the Table 1 , we find that the damping coefficient of the function control Buck regulator is small. It can be increased by feedingback the derivative of the output voltage . Taking the Buck regulator as an example, we may choose the function relation of the control circuit as follows:

$$d = \frac{K(V_r - v_0) - K_1n\,dv_0/dt}{v_g} \tag{24}$$

where n is the time constant, K_1 is the gain. The negative sign in front of $K_1n\,dv_0/dt$ denotes negative feedback. In this circumstances, the steady-state output voltage is the same as eq. (22) , and the small signal transfer function is:

$$\hat{v}_0= \frac{0\cdot\hat{v}_g+ (sL_dV_0/R^2)\hat{r}}{L_dCs^2+ [L_d/R + K_1n/(1 + K)]s + 1} \tag{25}$$

By selecting the value of K_1 and n , proper damping can be achieved to improve the dynamic behaviour . Similarly, the dynamic characteristics of other regulator can also be improved by feedingback dv_0/dt and the details are omitted here.

IV. FUNCTION CONTROL IMPLEMENTATION

It can be known from eq. (10) and Table 1 that the function control circuit can be implemented by two adding and substracting operational amplifiers, a divider and a modulator to turn the voltage into duty ratio. In practice application,a special function generator (SFG) shown in Fig.7 is used instead of the divider and the modulator . Its operating principle is similar to that obtained in [7],but it is more suitable here. In Fig. 7 , I_A and I_B are constant current sources and e_1 , e_2 are inputs. If $C_A/C_B= I_A/I_B$, the duty ratio of the special function generator is:

$$d = \frac{e_1}{e_1+ e_2} \tag{26}$$

The steady-state and the perturbation of duty ratio are as follows:

$$D = \frac{E_1}{E_1+ E_2} \tag{27}$$

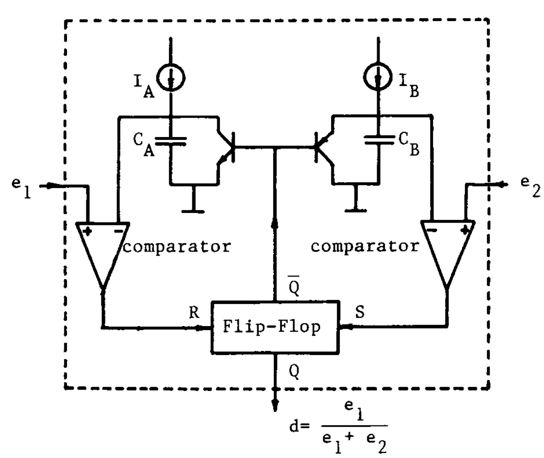

Fig. 7. Special function generator

$$\hat{d} = \frac{E_2\hat{e}_1 - E_1\hat{e}_2}{(E_1 + E_2)^2} \tag{28}$$

where $e_1 = E_1 + \hat{e}_1$, $e_2 = E_2 + \hat{e}_2$.

Here it should be noticed that because the capacitors C_a and C_b are charged with constant current sources,the poles and zeroes associated with them do not appear in the transfer function of special function generator.

On the basis of the above analysis , a unified block diagram of the function control switching regulator can be got, as shown in Fig. 8 . The function control circuit consists of operational amplifiers and a special function generator. The SFG is unchanged with various topologies of power stage. While the connection of amplifiers may be deduced from Table 1 and eq. (26),as illustrated in Table 2.

Table 2. Connection of amplifiers in four basic switching regulators

	Buck	Boost	Buck-Boost	Cuk
e_1	$K(V_r - v_0)$	$i_L - e_2$	$i_L - e_2$	$K(V_r - v_0)$
e_2	$v_g - e_1$	$K(V_r - v_0)$	$K(V_r - v_0)$	$v_{C1} - e_1$

Therefore, the function control circuit is very simple and very flexible. One control circuit can handle different types of power stage only with a change in the connection of operational amplifiers according to Table 2 and the engineering design of a switching regulator is greatly simplified.

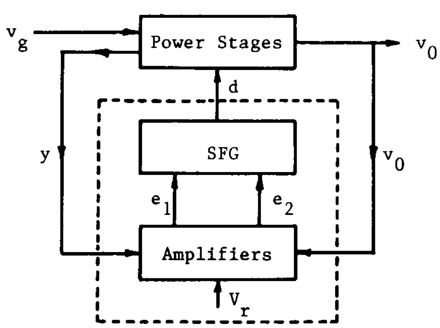

Fig. 8. Unified block diagram of function control switching regulator

V. EXPERIMANTAL VERIFICATION

A function control Buck regulator (Fig. 9) is breadboarded. The special function generator is the same as in Fig. 7 . R_1, R_2 are ESR of the output capacitors. R_3 is put there to get the derivative of the output voltage. When the line voltage v_g is 20 volts, the switching frequency is about 20kHZ , which is changed from about 16kHZ to 25kHZ when v_g varies from 26 volts to 16 volts. The output ripple voltage is about $20mV_{p-p}$ with $v_g = 20V$.

Table 3 is the measured audiosusceptibility = $20\log(\hat{v}_0/\hat{v}_g)$. Because of the second-order effects and other non-ideality, the audiosusceptibility can not achieve negative infinite as predicted by the theory. It is still very large.

Table 4 gives the measured output impedance Z_{om} and the calculated one ($Z_{oc} = -\hat{v}_0(s)/\hat{i}_0(s)$).Although the high frequency losses of the inductor, L , lead to a corresponding high equivalent series resistance ($R_1 = 7.57$ ohm), the measured maximum output impedance (0.162 ohm) is very low [8]. If better ferrite core and winding are used, much lower output impedance could be achieved. High ESR of the output capacitor results in the fact that the voltage accross the R_3 can not be considered proportional to dv_0/dt , but as follows:

$$\hat{v}_1 = \frac{sR_3C_2}{1 + s(R_2 + R_3)C_2}\hat{v}_0 \tag{29}$$

Eq. (29) is taken into account in calculating the output impedance Z_{oc}.

Fig. 10 is the transient response of the output voltage when the line voltage is abruptly switched from 20V to 24.5V, with a rise time about 40 microsecond. An MS-1650B digital memory oscilloscope is used as the transient recorder. The peak derivation is about 26mV. So the effect of the line voltage disturbance is reduced as much as 173 (= 4.5V/26mV) times.

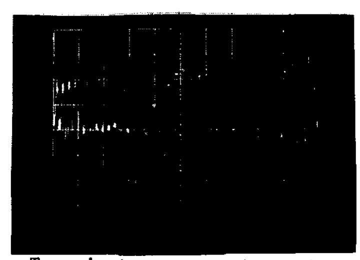

Fig. 10. Transient response to a step change of v_g (from 20V to 24.5V)
Vertical: 10mV/div., horizontal: 0.2ms/div.

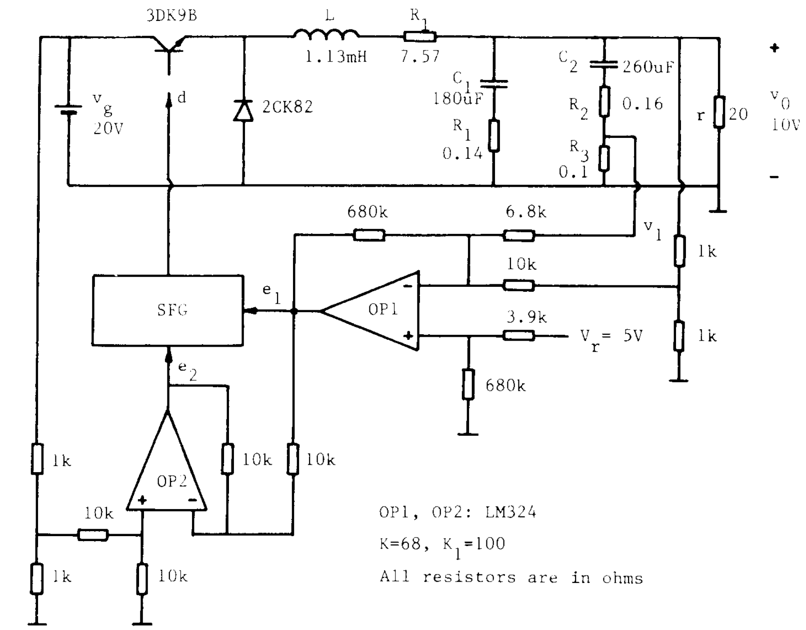

Fig. 9. Experimental model of function control Buck regulator

Table 3. Measured audiosusceptibility

f (HZ)	100	300	500	1000	1500	2000	3000	4000	5000	6000
audio. (db)	-60.4	-52.7	-49.7	-45.4	-43.7	-42.4	-41.9	-43.7	-45.2	-47.0

Table 4. Measured and calculated output impedance

f (HZ)	100	500	1000	1500	2000	2500	3000	4000	5000	6000
Z_{om} (ohm)	0.078	0.097	0.110	0.136	0.162	0.162	0.162	0.143	0.138	0.113
Z_{oc} (ohm)	0.110	0.119	0.137	0.152	0.164	0.174	0.181	0.179	0.165	0.150

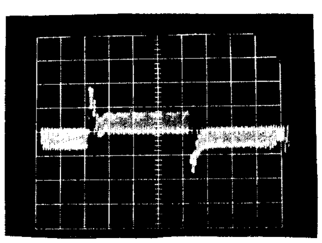

Fig. 11. Transient response to step change of load resistance (from 20 ohm to 40 ohm) Vertical: 20mV/div., horizontal: 0.5ms/div.

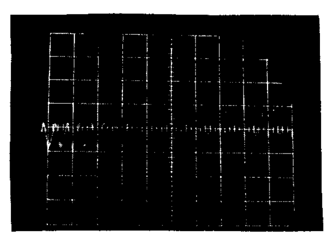

Fig. 12. Same as **Fig.11. , but the** horizontal scale is 0.1ms/div.

When the load resistance is switched repetively between r =20 ohm and r = 40 ohm, the response of the output voltage is given in Fig. 11 . The peak derivation is about 36mV and from Fig.12 , the output voltage returns to the regulated value in only about 4 switching periods, i.e. the transient duration is about 0.2ms. The test shows that fast response can be obtained by function control.

Although the analysis in the paper is made when the switching convertor operates in the continuous conduction mode, the output voltage keeps unchanged when it is in discontinuous conduction mode with the load resistance r = 180 ohm .

VI. CONCLUSION

The control strategy of dc-dc switching convertor is analysed in order to obtain a well-behaved switching regulator. It is studied in view of the function relation between the inputs and output of the control circuit. The goal of control strategy study is to select the appropriate feedback and feedforward and to formulate the function relation so as to get a high performance switching regulator.

The steady-state and small signal dynamic behaviours of four basic switching regulator by the function control are analysed and a simple implementation of function control circuit is presented, and as an example, a function control Buck regulator is breadboarded to confirm the analysis.

By the function control, the switching regulator is inherently stable, and the impacts of the disturbances from the line voltage and load resistance can be greatly reduced.The function control circuit is very simple and flexible, and the design of a function control switching regulator is much simplified and is very convenient.

Table 5 gives some comparison of the main characteristics among duty ratio cntrol, current programming control and function control.

In one word, the function control can greatly improve the main characteristics of a switching regulator in a very simple way. But it must be borne in mind that the function relation expressed in eq. (10) is not the unique one and more circuit variables of the power stage $(x_1, x_2, \ldots, x_n)$ might be fedback into the control circuit in order to make the performance of a switching regulator much higher.

Table 5. Comparison among duty ratio-,current programming- and function control switching regulator

	duty ratio	current programming	function
stability	poor	good	excellent
audiosusceptibility	poor	good	excellent
output impedance	poor	good	good
control simplicity	middle	complcated	simple
control flexibility	poor	poor	flexible
design simplicity	complicated	complicated	simple

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