SLIDING-MODE CONTROLLED CUK SWITCHING REGULATOR WITH FAST RESPONSE AND FIRST-ORDER DYNAMIC CHARACTERISTIC

Shi-Peng Huang, Hua-Qing Xu@, Yan-Fei Liu

Power Electronics Group, E. E. Dept.
Zhejiang University, Hangzhou, China

ABSTRACT

Under some constraints, the SMC is successfully applied to Cuk switching converter to get a fast response and first-order dynamic characteristic, with extremely large ability to depress line voltage and load current variations, as well as non-pulsating input and output currents. The theoretical analyses are in excellent agreement with experiments.

I. INTRODUCTION

In addition to the well-known outstanding properties, an optimal switching regulator should meet the following demands: (a) non-pulsating input and output currents; (b) extremely strong ability to suppress the line voltage disturbance; and (c) extremely strong ability to suppress the load current disturbance.

Several papers [1,2] have been made to get an optimal switching regulator. The current-programmed control has been developed to overcome the drawbacks of the widely used duty-ratio control and by properly feeding-forward the line voltage and load current, the current-programmed controlled Buck, Boost and Buck-Boost switching regulators can eliminate the impacts caused by disturbances of both line voltage and load current[1]. But the analyses and synthesis are very complicatedand its implementation of control circuit is complicated, too. The function control[2] has been applied to DC-DC switching converters and these switching regulators satisfy demands (a) and (b), however demand (c) can not be met, yet. In addition, the existing controls are all based on the small-signal concept, thus their conclusions are not valid for large-signal variations.

With its peculiar properties, the sliding-mode control(SMC) technique has been applied to the Buck, Boost and Buck-Boost converters and obtained highly performanced switching regulators with first-order response and large ability to suppress disturbances[3]. But it has not been introduced to tackle the Cuk converter.

Cuk converter, as shown in Fig. 1, is superior to other converters on its non-pulsating input and output currents, either step-up and step-down voltage gain, and so on [4]. Nevertheless, it is a fouth-order system with poor open-loop dynamic characteristic because of the right-half-plane zeroes.

It is therefore benificial to construct a switching regulator which keeps the merits of both SMC and Cuk converter, while eleminate their demerits.

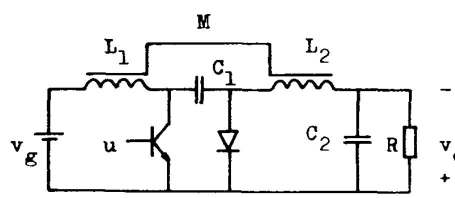

Fig. 1 Cuk converter with L_1 L_2 coupled

In the Fig. 1, u is a discontinuous control variable with its value equals to 1 or 0.

@ Hua-Qing Xu is now with Fuzhou University, Fuzhou, China

After reviewing the principle of SMC in section II, in section III the sliding-mode controlled Cuk switching regulator is analysed. The predicted first-order and fast response characteristics are verified in section IV.

II. PRINCIPLE OF SLIDING-MODE CONTROL

The theory of variable structure system and sliding-mode control are time-domain analysis- and synthsis-technique. It can be used to characterize the system under both small- and large-signal conditions. The theory has been described in detail in [3]. A brief review is made here.

Consider a single-input and single-output non-linear system represented in the following form:

$$\dot{Y} = AY + Bu + C$$
$$Y = \left[y_o, \frac{dy_o}{dt}, \frac{d^2y_o}{dt^2}, \ldots, \frac{d^{n-1}y_o}{dt^{n-1}}\right]^T \qquad (1)$$

where Y represents the system output voltage error and its successive derivative matrix, $\dot{Y}$ the first-order derivative of Y. A, B and C are system parameter matrices, u is a discontinuous control variable u = 1 or u = 0.

According to eq.(1), a new function or a new equation can be constructed as follows:

$$\sigma = GY = g_0 y_o + g_1 \frac{dy_o}{dt} + \ldots + g_{n-1}\frac{d^{n-1}y_o}{dt^{n-1}} = 0 \qquad (2)$$

$$G = [g_0, g_1, \ldots, g_{n-1}]$$

The new function $\sigma = GY$ is the weighted sum of the output error y_o and its successive derivatives. The elements of matrix G are the feedback gains of y_o and its derivatives. In geometry, the new equation $\sigma = 0$ denotes a hyperplane in the n-dimensional space whose axes are the output error and its successive derivatives.

The principle of sliding-mode control can be expressed as that, by the discontinuous control variable u, the operating point of the system (y_o, $dy_o/dt, \ldots, d^{n-1}y_o/dt^{n-1}$) is constrainted on the hyperplane described by eq.(2). Therefore, the dynamic behaviour under SMC is solely determined by eq.(2). In other words, eq.(2) expresses the dynamic characteristics of the system.

To complete the sliding-mode control, the "reaching condition" and the "existing condition" must be satisfied. The reaching condition is that the system is able to reach the sliding plane $\sigma = GY = 0$ from any arbitrary initial operating point under the input control u. This condition will be interpreted by phase graph. The "existing condition" refers to that once arriving at the hyperplane, the operating point is able to be kept on the sliding surface.

Mathematically, the "existing condition" can be expressed as

$$\lim_{\Delta t \to 0} \frac{\Delta\sigma}{\Delta t} = \dot{\sigma} < 0 \quad \text{when } \sigma > 0$$
$$\lim_{\Delta t \to 0} \frac{\Delta\sigma}{\Delta t} = \dot{\sigma} > 0 \quad \text{when } \sigma < 0 \qquad (3)$$

If we assume that:

$$u = u^+ \quad \text{when} \quad \sigma > 0$$
$$u = u^- \quad \text{when} \quad \sigma < 0 \qquad (4)$$

Combining eqs.(1), (2), (3), (4), the following relation can be deduced to meet the existing condition:

$$GAY + GBu^+ + GC < 0 < GAY + GBu^- + GC \qquad (5)$$

This is the so-called "control strategy". If the input control u is properly chosen to be 1 or 0 according to eq.(5), the dynamic characteristics of the system are described by $\sigma = 0$, or by the feedbackgains G, being independent on other parameters of the system. It is one merit of the sliding-mode controlled system and is called "robustness".

Another merit of SMC is the order-reduction of the system. By SMC, the dynamic behaviour of a n-order system with m (m<n) input controls can be described by a (n-m)-order differential equation. For example, the DC-DC switching converter is usually a second-order system with a input control, the duty ratio, its closed-loop dynamic behaviour by SMC is a first-order system with inherent stability, no overshoot and first-order response [3,5].

The third merit of SMC is that the dynamic equation (2) of the system is a linear differential equation reconstructed out from the nonlinear equation of the switching converter without the use of linear ized technique.

III. SLIDING-MODE CONTROLLED CUK SWITCHING REGULATOR

The Cuk converter shown in Fig. 1 can be described by the following equations:

$$L_2C_2(1-k^2)\frac{d^3v_o}{dt^3} + \frac{L_2}{R}(1-k^2)\frac{d^2v_o}{dt^2} + (1+\frac{C_2}{C_1})\frac{dv_o}{dt} + \frac{1}{RC_1}v_o = 0 \quad \text{when } u = 1 \qquad (6a)$$

$$L_2C_2(1-k^2)\frac{d^4v_o}{dt^4} + \frac{L_2}{R}(1-k^2)\frac{d^3v_o}{dt^3} + (1+\frac{L_2C_2}{L_1C_1})\cdot\frac{d^2v_o}{dt^2} + \frac{L_2}{L_1RC_1}\frac{dv_o}{dt} + \frac{v_o}{L_1C_1} = 0 \quad \text{when } u=0 \qquad (6b)$$

where k is the coupled-coefficient between L_1 and L_2, $k = M/\sqrt{L_1L_2}$, u is the discontinuous input control, u = 1 corresponds to the conduction of transistor and u = 0 to that of diode.

It can be seen that the Cuk converter is a fourth-order circuit with one input control. If the SMC technique is directly applied to it, a third-order closed-loop system is obtained, with poor dynamic property difficult to grasp.

The right half part of Cuk converter shown in the dotted block in Fig.3 can be regarded as a Buck topology with its equivalent input voltage v_{C1}. From the point of view, if the value of C_1 is chosen to be much larger than that of C_2, i.e., $C_1 \gg C_2$, the variation of the voltage across C_1 is much smaller than that across C_2, so that the voltage v_{C1} can be considered constant within one switching period, i.e. v_{C1}= const. The following equations can be deduced:

$$L_2C_2(1-k^2)\frac{d^2v_o}{dt^2} + \frac{L_2}{R}(1-k^2)\frac{dv_o}{dt} + v_o = v_{C1} - \frac{M}{L_1}v_g \quad \text{when } u = 1 \qquad (7a)$$

$$L_2C_2(1-k^2)\frac{d^2v_o}{dt^2} + \frac{L_2}{R}(1-k^2)\frac{dv_o}{dt} + v_o = \frac{M}{L_1}(v_{C1}-v_g) \quad \text{when } u = 0 \qquad (7b)$$

Eq.(7) shows that under the constraint of $C_1 \gg C_2$, Cuk converter can be described by a set of second-order differential equations.

The reaching condition and the existing condition of SMC will be analysed in the circumstances of k = 0, $0 < k < 1$, and k = 1.

1 If k = 0, from eq.(7):

$$L_2C_2\frac{d^2v_o}{dt^2} + \frac{L_2}{R}\frac{dv_o}{dt} + v_o = v_{C1} \quad \text{when } u=1$$
$$L_2C_2\frac{d^2v_o}{dt^2} + \frac{L_2}{R}\frac{dv_o}{dt} + v_o = 0 \quad \text{when } u=0 \qquad (8)$$

Fig. 2 gives the phase-plane description of Cuk converter where $v_{C1}-V_o^* > 0$, V_o^* is the reference voltage.

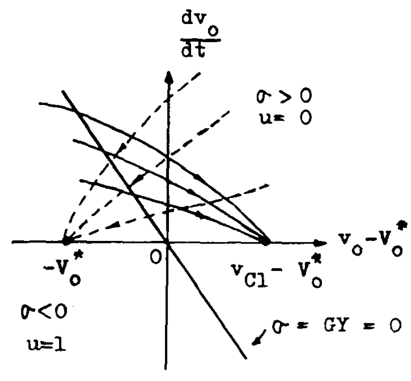

Fig. 2 The phase-plane description when k = 0

It can be seen that from any initial state of the system, the operating point can reach its steady-state $v_{C1}-V_o^*$ or $-V_o^*$ when u = 1 or u = 0, as shown by the solid curve and the dotted one, respectively. According to the theory of VSS with SMC, the new equation describing the dynamic behaviour can be deduced as follows:

$$\sigma = GY = 0 \qquad (9)$$

$$Y = [\, v_o - V_o^*,\ \frac{dv_o}{dt}]^T, \quad G = [\, g_0,\ g_1]$$

or

$$\sigma = g_0(v_o - V_o^*) + g_1\frac{dv_o}{dt} = 0 \qquad (10)$$

Eq.(10) is the sliding line in the phase-plane. The phase-trajetories of of eq.(8) will always hit the sliding line , whenever u = 1 or u = 0. Thus, satisfied is the reaching condition of sliding-mode control. The existing condition can easily be met by choosing the input control u as:

$$u = u^+ = 0 \quad \text{when } \sigma > 0$$
$$u = u^- = 1 \quad \text{when } \sigma < 0 \qquad (11)$$

2 If $0 < k < 1$

(a) If $\frac{M}{L_1} \geq 1$, with following constraints:

$$v_{C1} - \frac{M}{L_1} v_g - V_o^* \leq 0$$
$$\frac{M}{L_1}(v_{C1} - v_g) - V_o^* \geq 0 \qquad (12)$$

The phase-plane description of eq.(7) is given in Fig. 3(a).

(b) If $\frac{M}{L_1} \leq 1$, with following constraints:

$$v_{C1} - \frac{M}{L_1} v_g - V_o^* \geq 0$$
$$\frac{M}{L_1}(v_{C1} - v_g) - V_o^* \leq 0 \qquad (13)$$

The phase-plane description of eq.(7) is drawn in Fig. 3(b).

From Fig. 3 , when $0 < k < 1$, in the cases of both $\frac{M}{L_1} \geq 1$ and $\frac{M}{L_1} \leq 1$, the reaching condition of SMC is met,

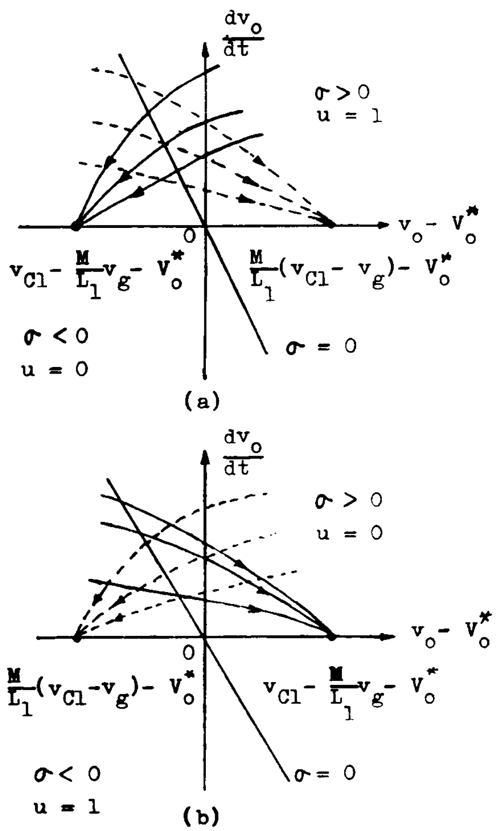

Fig. 3 The phase-plane descriptions when $0<k<1$, (a) $M/L_1 \geq 1$, (b) $M/L_1 \leq 1$

While the control strategy to meet the existing condition should be chosen as follows:

(a) in the case of $\frac{M}{L_1} \geq 1$

$$u = u^+ = 1 \quad \text{when } \sigma > 0$$
$$u = u^- = 0 \quad \text{when } \sigma < 0 \qquad (14)$$

and

(b) in the case of $\frac{M}{L_1} \leq 1$

$$u = u^+ = 0 \quad \text{when } \sigma > 0$$
$$u = u^- = 1 \quad \text{when } \sigma < 0 \qquad (15)$$

Therefore, when $0 < k < 1$, the SMC can also be used to handle the Cuk converter for obtaining a first-order dynamic response, however, the constraints are more critical.

3 If k = 1, eq.(7) is simplified as:

$$v_o = v_{C1} - \frac{M}{L_1} v_g \quad \text{when } u=1$$
$$v_o = \frac{M}{L_1}(v_{C1} - v_g) \quad \text{when } u=0 \tag{16}$$

It is obvious that the output of the system is fixed on the two steady state points, and there are no phase trajetories. Therefore it can not be applied the SMC to this case.

The analyses above are summarized in Table 1.

IV. IMPLEMENTATION OF SMC CUK REGULATOR AND EXPERIMENTAL RESULTS

For simplicity, Cuk converter without the coupling, as shown in Fig.4 was breadboarded. The derivative of v_o is obtained by sensing the current in C_2 according to the equation $i_{C2} = C_2(dv_o/dt)$. Therefore the sliding line is:

$$\sigma = K_1(v_o - V_o^*) + K_2 Rs\ C2 \frac{dv_o}{dt} = 0 \tag{17}$$

where K_1 is the error voltage feedback gain, K_2 is the feedback gain of current i_{C2} and R_s is the current sensing resistance.

In Fig.4 the sliding line formation network consists of adding, substracting Op. Amp. while the driver consists of the Schmitt trigger and the pulsed amp.

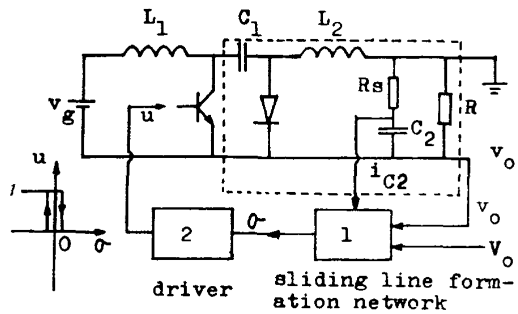

Fig. 4 Block diagram of SMC Ćuk regulator

Table 1.

coupled coefficient of L_1 and L_2	$\frac{M}{L_1}$	steady-state point of v_o		constrants besides $C_1 \gg C_2$	reaching condition satisfied or not	SMC strategy
		u=1	u=0			
k = 0		v_{C1}	0	$v_{C1} > V_o^*$	yes	$u=u^+=0$, if $\sigma>0$ $u=u^-=1$, if $\sigma<0$
0< k< 1	$\geqslant$ 1	$v_{C1} - \frac{M}{L_1} v_g$	$\frac{M}{L_1}(v_{C1} - v_g)$	$v_{C1} - \frac{M}{L_1} v_g \leqslant V_o^*$ $\frac{M}{L_1}(v_{C1} - v_g) \geq V_o^*$	yes	$u=u^+=1$ if $\sigma>0$ $u=u^-=0$ if $\sigma<0$
	$\leqslant$ 1			$v_{C1} - \frac{M}{L_1} v_g \geq V_o^*$ $\frac{M}{L_1}(v_{C1} - v_g) \leqslant V_o^*$	yes	$u=u^+=0$ if $\sigma>0$ $u=u^-=1$ if $\sigma<0$
k = 1					no	no

The steady-state and dynamic characteristics of the sliding-mode controlled Cuk regulator are determined solely by the linear differential eq.(17).

From the analyses in the last section,when the operating point has hit the sliding line, it will be constrainted at the sliding line. In other words, eq.(17) gives a total description of sliding-mode controlled Cuk regulator. It is valid for both small- and large-signal disturbances.The output voltage does not vary with the disturbances of both line voltage and load current. In addition it is a first-order system with its time constant ($K_2 R_s C_2/K_1$) chosen by the designer to get the fast response.

Experiments show excellent performances of Cuk regulator by SMC. When the line voltage steps up, the variation of output voltage is given in Fig. 5 while Fig. 6 shows its response to a step change of load current.

In the practical system, due to the hysteristic in the Schmitt trigger, the steady-state voltage has an error. Besides, the ripple voltage is high.

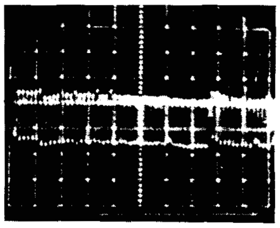

Fig. 5 Response to line voltage step change, ver. 0.1V/div. hor. 0.5ms/div

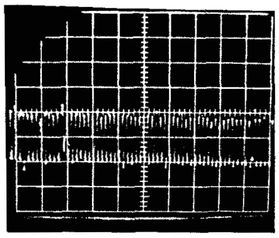

Fig. 6 Response to load current step change ver. 0.1V/div. hor. 0.5ms/div.

The photographs illustrate that the output voltage v_o is very insensive to disturbances and it returns to its original value very quickly.

V. CONCLUSIONS

According to the features of Cuk converter and the basic principle of the sliding-mode control, the SMC is successfully applied to Cuk converter. It is a highly-performanced switching regulator with fast response and first-order dynamic characteristic and with extremely strong ability to suppress both the line voltage and the load current disturbances of both small- and large-signals, as well as with non-pulsating input and output currents. The performance of SMC Cuk regulator is determined by feedback gains.

In the light of the new concept of control, other high-order power stages, such as the augmented Boost and augmented Buck converters[6] and the Buck converter with input filter, can also be handled by SMC to get a highly-performanced closed-loop system.

REFERENCES

[1] R. Redl, at el: "Near-Optimal Dynamic Regulation of DC-DC Converters Using Feedforward of Output Current and Input Voltage with Current-Mode Control", IEEE PE-1, No.3, July 1986

[2] Yan-Fei Liu, at el: "Function Control and Its Application to DC-DC Switching Regulators", IEEE PESC 1988

[3] R. Venkataramanan, at el: "Sliding-Mode Control of DC-DC Converter" IECON 1985

[4] R. D. Middlebrook, at el: "Isolation and Multiple Output Extensive of a New Optimum Topology Switching DC-to-DC Converter", IEEE PESC 1978

[5] F. Bilalovic, at el: "Buck Converter Regulator Operating in the Sliding Mode", PCI 1983 Proceding

[6] G. C. Hua, at el: "New PWM Switched-Mode Converter Topologies", IEEE PESC 1988