A General Unified Large Signal Model for Current Programmed DC-to-DC Converters

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***Abstract*—A general and unified large signal averaged circuit model for current programmed DC-to-DC converters is proposed. In the averaged circuit model, the active switch is modeled by a current source, with its value equal to the averaged current flowing through it, and the diode is modeled by the voltage source, with its value equal to the averaged voltage across it. The averaged circuit model has the same topology as the switching converter. The large signal averaged circuit model for current programmed Buck, Boost, Buck-Boost and Ćuk converters are proposed, from which the large signal characteristics can be obtained. The steady-state and small signal transfer functions of the current programmed DC-to-DC converters can all be derived from their large signal averaged circuit models. The large signal characteristics of current programmed Buck converter are studied by both the phase plane trajectory and the time domain analysis. Experimental prototypes for a current programmed Buck converter, with and without an input filter, are breadboarded to verify the analysis.**

I. Introduction

The current programmed control is becoming widely used in the power supply field because of its advantages over the conventional direct duty ratio control, such as fast response, improved damping, automatic over current protection, and current sharing [1], [2]. Unfortunately, because there are two feedback loops in the current programmed control, the analysis of the dynamic characteristics of the current programmed converter is very complicated. Only the small signal models [3]–[5] are developed for the current programmed DC-to-DC converters. There is not even a unified model that can predict both the steady-state and small signal dynamic characteristics of the current programmed converter with an artificial stabilizing ramp.

The switching converters under current programmed control are pulsed and nonlinear dynamic system. Such a system may be stable in the vicinity of the operating point, but may not be stable when the system undergoes a large perturbation. Therefore, the small signal model cannot predict the stability information when the system is subjected to large signal perturbation or large parameter variation. A large signal model for the current programmed converters is essential to study the global dynamic characteristics of the switching converters under current programmed control and to design robust and high-performance switching power supply.

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The paper [6] investigates the large signal characteristics of current programmed converter. The approach used in [6] is qualitative in nature and is suitable only for the Boost converter. In addition, extensive mathematical derivation is needed to obtain the result. It does not provide the general form for large signal, small signal, and steady-state analysis.

In this paper, a unified general model is proposed to analyze the large signal characteristics of the DC-to-DC switching converters under the current programmed control at continuous conduction mode. This technique uses the state space average technique [7] and takes the form of the averaged circuit model that has the same topology as that of the switching converter under investigation. The following are the advantages of the proposed model.

1) *Simple:* The averaged circuit model has the same topology as the switching converters under investigation. No matter how complicated the topology is, the averaged circuit model can always be derived easily.
2) *General:* This technique can be applied to all current programmed DC-to-DC converters.
3) *Unified:* The small signal transfer functions and the steady-state input output relations can all be derived conveniently from the large signal averaged circuit model.
4) *Powerful:* The effect of the input filter, output filter, and parasitic parameters can all be considered with little difficulty.

In the next section, the large signal averaged circuit model of the switching converter under duty ratio control is at first derived. In Section III, the technique used to derive the general and unified large signal models for the current programmed converters is proposed, and the large signal averaged circuit model for current programmed Buck converter is derived as an illustration. The large signal averaged circuit models for current programmed Boost, Buck-Boost, and Ćuk converters, as well as the Buck converter with an input filter, are proposed in Section IV. In Section V, the large signal characteristics of the Buck converter under current programmed control is investigated. The small signal transfer functions are also derived, which are actually the same as those derived in [4]. The steady-state relation is also presented. In Section VI, the validity of the proposed averaged circuit model is verified experimentally by the current programmed Buck converter, with or without an input filter. Section VII is the conclusion.

II. Averaged Circuit Model at Duty Ratio Control

The output voltage of the switching converter is controlled by the duty ratio of the active switch. In the direct duty ratio control, the duty ratio is the control variable. In the current programmed control, the control variable is the peak current, and the duty ratio is controlled indirectly. Because of the close relationship between the direct duty ratio control and the current programmed control, it is worthwhile to derive the averaged circuit model of the DC-to-DC switching converters under direct duty ratio control.

The state space averaging method [7] has been applied successively to model the dynamic characteristics of the switching converter. Its basic idea is to study the behavior of the averaged model of the switching converter. In the averaged model, all the circuit variables are the averaged value of the actual circuit variables. In [7], this averaged model is expressed by a set of state space differential equations. Small signal transfer functions and the canonical circuit model [7] can be derived by perturbing this nonlinear averaged model.

According to the circuit theory, for every set of state space differential equations, a circuit topology can be found that has the same state space differential equations. This argument provides another possibility to express the averaged model, i.e., using the averaged circuit model, in which every circuit variable is the averaged value of the corresponding instantaneous variable. Take the Buck converter, Fig. 1(a), as an example to illustrate this concept. In Buck converter, the active switch (transistor) Q and the diode D are nonlinear components. The waveforms of the current flowing through the active switch, $i_Q(t)$, and the voltage across the diode, $v_d(t)$, are given in Fig. 1(b) at low ripple assumption. When the active switch is on, the inductor current flows through Q, and the diode voltage, $v_d(t)$, equals the supply voltage (assuming ideal switches). When the active switch is off, the inductor current flows through diode, and both $i_Q(t)$ and $v_d(t)$ are 0. Instead of taking average in the form of state space differential equations, the averaging process can be achieved directly on the switching converter itself. The switch Q can be modeled by a controlled current source, i_Q, and the diode can be modeled by a controlled voltage source, v_d. The value of i_Q equals to the averaged current flowing through Q, and the value of v_d is the same as the averaged voltage across the diode. The following are obvious, from Fig. 1(b):

$$i_Q = \frac{t_{\text{on}}}{T_s} i_L = \alpha i_L, \tag{1}$$

and

$$v_d = \frac{t_{\text{on}}}{T_s} v_s = \alpha v_s, \tag{2}$$

where t_{on} is the on time of the active switch, T_s is the switching period, and α denotes the duty ratio. The averaged model for the inductor, capacitor, resistor, and the supply voltage remains unchanged, because they are present for both the on and off states. The averaged circuit model is therefore obtained and is as shown in Fig. 1(c).

The validity of this averaged circuit model can be proved easily. The state space differential equations derived from the averaged circuit model, Fig. 1(c), are as follows:

$$\frac{di_L}{dt} = \frac{\alpha v_s - v_O}{L}, \tag{3a}$$

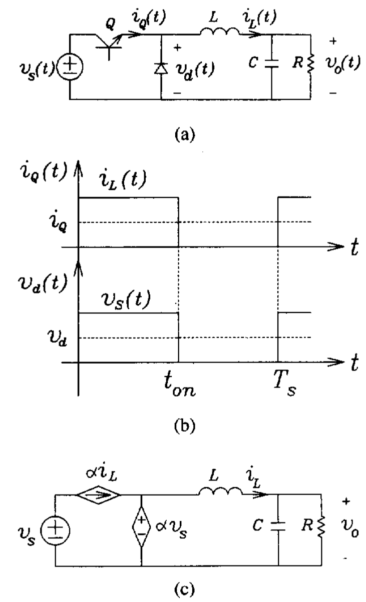


Fig. 1. Derivation of the averaged circuit model of Buck converter. (a) Buck converter. (b) Switching waveforms. (c) Averaged circuit model.

$$\frac{dv_O}{dt} = \frac{i_L - v_O/R}{C}. \tag{3b}$$

Equation (3) is the same as that derived from the state space averaged model [7]. Therefore, the averaged circuit model, shown in Fig. 1(c), can provide all the information that the state space averaged model [7] can. When the nonlinear expression of the switch current i_Q and diode voltage v_d are linearized around the operating point, the small signal transfer function can be derived from this large signal averaged circuit model. The results are the same as those obtained from the state space averaged model [8].

To summarize, the large signal averaged circuit model of the switching converter at duty ratio control can be obtained according to the following procedure.

1) Substitute the active switch by a controlled current source $i_Q = \alpha i_{Qp}$, where i_{Qp} is the current flowing through the switch when it is on.
2) Substitute the diode by a controlled voltage source $v_d = \alpha v_{dp}$, where v_{dp} denotes the voltage appeared across the diode when it is off.
3) All the other parts of the switching converter remain unchanged.

Using this technique, the large signal nonlinear averaged circuit models for Boost, Buck-Boost, and Ćuk converters, as shown in Fig. 2, can be obtained and are given in Fig. 3. Their validity can be verified easily by the fact that the state space differential equations derived from these models are exactly the same as those derived from the state-space averaged models, which have already been verified experimentally.

III. Basic Idea of the Averaged Circuit Model for Current Programmed DC-to-DC Converters

In the current programmed control [1], the switch is turned on by clock and is turned off when the inductor current reaches

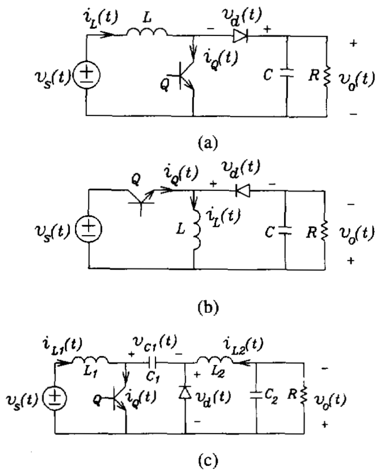


Fig. 2. Other switching converters. (a) Boost. (b) Buck-Boost. (c) Ćuk converter.

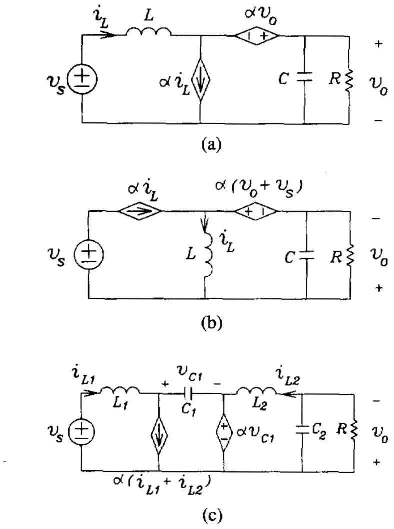


Fig. 3. Averaged circuit model for other converters. (a) Boost. (b) Buck-Boost. (c) Ćuk converter.

a threshold value determined by the control signal. Therefore, this threshold value now becomes the control variable to the switching converter. In this case, the duty ratio is only controlled indirectly and can be expressed by other circuit variables.

Fig. 4 gives the actual detailed relationship between the control signal, i_c, and the inductor current, $i_L(t)$ [3]. An artificial ramp with slope $-M$ is also included to avoid the subharmonic oscillation. In Fig. 4, $i_L(t)$ denotes the instantaneous inductor current, and i_L denotes the state-space averaged inductor current. Because the state-space averaged inductor current i_L passes through the midpoint of the actual current waveform, the following is clear from geometry:

$$i_L = i_c - \frac{1}{2} m_1 \alpha T_s - \alpha M T_s, \tag{4}$$

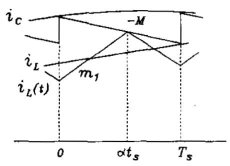


Fig. 4. Detailed waveforms between inductor current i_L and control signal i_c in current programmed control.

where m_1 is the slope of the rising segment of the inductor current, $-M$ is the slope of the artificial ramp, and T_s is the switching period. Equation (4) gives the relation between the control signal i_c, averaged inductor current i_L, the duty ratio α, and some other circuit variables.

Equation (4) is true only for the steady-state condition. A modified expression that considers the dynamic quantities is proposed in [10]. However, (4) is used in the following analysis as an approximation, because (a) it is shown in [11] that the small signal dynamic models derived from (4) and from the modified expression proposed in [10] are very close, and (b) by using (4), the derivation is simplified and the procedure to derive the large signal model can be illustrated more clearly.

In the current programmed control, the duty ratio α is no longer the control variable, but can be expressed from (4) as follows:

$$\alpha = \frac{i_c - i_L}{\frac{1}{2} m_1 T_s + M T_s}. \tag{5}$$

Equation (5) holds true for all switching converters. For different converters, only the expression for m_1 is different. For example, in Buck converter, m_1 can be expressed as follows:

$$m_1 = \frac{v_s - v_o}{L}, \tag{6}$$

where L is the value of the filter inductor.

Recall that (a) it is the behavior of the averaged value of the circuit variables that are most interesting, and (b) the active switch can be modeled by a controlled current source, with its value equal to the averaged current flowing through it, and the diode can be modeled by a controlled voltage source, with its value equal to the averaged voltage across the diode. Therefore, the objective to derive the large signal model for the current programmed converters now becomes to find the averaged switch current, i_Q, and the averaged diode voltage, v_d, of the current programmed converters.

Take the Buck converter as an example. Its averaged circuit model is given in Fig. 1(c). In the current programmed control, the duty ratio α is no longer independent, but is controlled by the control signal i_c and some other circuit variables, and can be expressed from (5) and (6) as follows:

$$\alpha = \frac{i_c - i_L}{\frac{T_s}{2L}(v_s - v_o) + M T_s}. \tag{7}$$

Using (7), the averaged current flowing through the active switch i_Q and the averaged voltage across the diode v_d at the

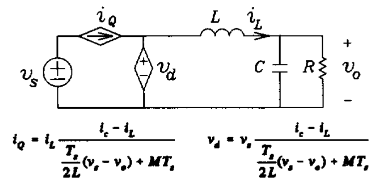


Fig. 5. Large signal averaged circuit model for current programmed Buck converter $i_Q = i_L \frac{i_c - i_L}{\frac{T_s}{2L}(v_s - v_o) + MT_s}$, $v_d = v_s \frac{i_c - i_L}{\frac{T_s}{2L}(v_s - v_o) + MT_s}$.

current programmed control can be found as follows:

$$i_Q = \alpha i_L = i_L \frac{i_c - i_L}{\frac{T_s}{2L}(v_s - v_o) + MT_s}, \tag{8}$$

$$v_d = \alpha v_s = v_s \frac{i_c - i_L}{\frac{T_s}{2L}(v_s - v_o) + MT_s}. \tag{9}$$

It is noticed that the current i_Q and the voltage v_d are not independent, but are controlled by other circuit variables, such as i_c, i_L, v_s, v_o, and so forth. When transistor and diode are substituted by their averaged current (8) and averaged voltage (9), respectively, a large signal averaged circuit model for current programmed Buck converter is obtained, as shown in Fig. 5.

Because all of the circuit variables are state-spaced averaged value, and because no small signal assumption is imposed during the above derivation, the model given in Fig. 5 is a large signal model for the current programmed Buck converter. In this model, the current loop is absorbed, and the duty ratio is no longer present. The control variable to the power stage is the current command, i_c. The active switch is modeled by the controlled current source, i_Q, which is equal to the averaged current flowing through it, and the diode is modeled by the controlled voltage source, v_d, which equals the averaged voltage across the diode. The other parts of the averaged circuit model are the same as those of the Buck converter.

From the above analysis, the procedure used to derive the large signal averaged circuit model for current programmed converters can be summarized as follows.

1) Derive the large signal averaged circuit model of the switching converter under direct duty ratio control according to the procedure developed in the previous section.
2) Find the expression of the rising slope of the inductor current m_1 for the power stage under investigation, e.g., (6).
3) Express the duty ratio α by the control signal, i_c, inductor current i_L, and some other circuit variables, e.g., (7).
4) Substitute the duty ratio α into the averaged circuit model established in step (1) by the expression found in step (3) to obtain the averaged current flowing through the active switch, i_Q, and the averaged voltage across diode, v_d.
5) Replace the active switch by the controlled current source i_Q and the diode by the controlled voltage source v_d.
6) All of the other parts of the power stage remain unchanged.

Following this procedure, the large signal nonlinear averaged circuit model for the current programmed converters can be obtained.

Once the large signal model is established, the large signal characteristics can be analyzed. One way to do so is to use the state-space differential equations. For the Buck converter, the state-space differential equations can be derived from its averaged circuit model (Fig. 5) as follows:

$$L\frac{di_L}{dt} = v_s \frac{i_c - i_L}{\frac{T_s}{2L}(v_s - v_o) + MT_s} - v_o, \tag{10a}$$

$$C\frac{dv_o}{dt} = i_L - \frac{v_o}{R}. \tag{10b}$$

By integrating (10), the response of the averaged inductor current and output voltage can be calculated, and, consequently, the large signal dynamic characteristics can be investigated.

Another way to do this is to use a circuit simulation software package, e.g., PSPICE, to obtain the large signal dynamic characteristics, because the averaged circuit model of Fig. 5 can be entered directly into such a package. The designer needs to specify only the configuration of the circuit, its parameters, and the type of analysis, and the software can formulate the circuit equations and give the corresponding results.

The advantage of the proposed model is that the mathematical derivation and the effort to obtain the large signal dynamic characteristics of the current programmed converters are minimized, so that the designer can concentrate his effort on how to improve the performance to meet the specifications. Another advantage is that the effect of the parasitic parameters, such as winding resistor of the inductor, equivalent series resistor (ESR) of the filter capacitor, and so forth, can be evaluated easily when either of the method discussed above is employed to analyze the large signal characteristics.

It is noted that in the above derivation, the sample-and-hold effect associated with the current programmed control is not considered. Therefore, the derived large signal model is suitable only for the case when the supply voltage and control signal changes slowly; i.e., it is much slower as compared with the switching period.

IV. Averaged Circuit Model for Other Converters

In this section, the analysis of the previous section is extended, and the averaged circuit models for current programmed Boost, Buck-Boost, and Ćuk converters, as well as a Buck converter with an input filter, are derived based on the technique proposed in the previous section.

A. Current Programmed Boost Converter

The averaged circuit model for the Boost converter under the duty ratio control is given in Fig. 3(a). The inductor current rises when the switch Q is on. The slope m_1 can be expressed as follows:

$$m_1 = \frac{V_s}{L}. \tag{11}$$

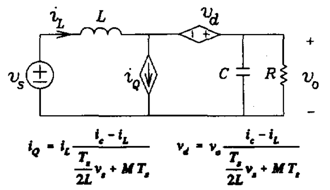


Fig. 6. Large signal averaged circuit model for current programmed Boost converter $i_Q = i_L \frac{i_c - i_L}{\frac{T_s}{2L}v_s + MT_s}$, $v_d = v_o \frac{i_c - i_L}{\frac{T_s}{2L}v_s + MT_s}$.

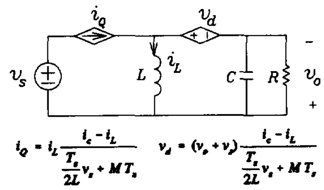


Fig. 7. Large signal averaged circuit model for current programmed Buck-Boost converter $i_Q = i_L \frac{i_c - i_L}{\frac{T_s}{2L}v_s + MT_s}$, $v_d = (v_o + v_s) \frac{i_c - i_L}{\frac{T_s}{2L}v_s + MT_s}$.

Combining (5) and (11), the duty ratio α can be expressed as follows:

$$\alpha = \frac{i_c - i_L}{\frac{T_s}{2L}v_s + MT_s}. \tag{12}$$

Therefore, the averaged current flowing through Q, i_Q, and the averaged voltage across diode, v_d, can be found as follows:

$$i_Q = \alpha i_L = i_L \frac{i_c - i_L}{\frac{T_s}{2L}v_s + MT_s}, \tag{13}$$

$$v_d = \alpha v_o = v_o \frac{i_c - i_L}{\frac{T_s}{2L}v_s + MT_s}. \tag{14}$$

In the Boost converter, Fig. 2(a), replacing the switch Q by the controlled current source i_Q (13), and the diode with the controlled voltage source v_d (14), the large signal averaged circuit model for current programmed Boost converter is obtained, as shown in Fig. 6. The large signal dynamic characteristics can be analyzed either by importing the averaged circuit model into PSPICE or by integrating the state space differential equations derived from the large signal model (Fig. 6) as follows:

$$L\frac{di_L}{dt} = v_s - v_o\left(1 - \frac{i_c - i_L}{\frac{T_s}{2L}v_s + MT_s}\right), \tag{15a}$$

$$C\frac{dv_o}{dt} = i_L\left(1 - \frac{i_c - i_L}{\frac{T_s}{2L}v_s + MT_s}\right) - \frac{v_o}{R}. \tag{15b}$$

B. Current Programmed Buck-Boost Converter

The averaged circuit model for Buck-Boost converter at duty ratio control is shown in Fig. 3b. The rising slope of the inductor current for Buck-Boost converter is the same as that of Boost converter. Therefore, the expression for the duty ratio α is also the same as that for Boost converter, given by (12). The averaged switch current i_Q and averaged diode voltage v_d can be expressed as follows:

$$i_Q = \alpha i_L = i_L \frac{i_c - i_L}{\frac{T_s}{2L}v_s + MT_s}, \tag{16}$$

$$v_d = \alpha(v_o + v_s) = (v_o + v_s)\frac{i_c - i_L}{\frac{T_s}{2L}v_s + MT_s}. \tag{17}$$

In a similar way, the large signal averaged circuit model of Buck-Boost converter under current programmed control can be obtained by replacing the active switch and the diode with the controlled current source i_Q (16), and controlled voltage source v_d (17), respectively, as shown in Fig. 7. The state space differential equations can be derived from Fig. 7 as follows:

$$L\frac{di_L}{dt} = (v_o + v_s)\frac{i_c - i_L}{\frac{T_s}{2L}v_s + MT_s} - v_o, \tag{18a}$$

$$C\frac{dv_o}{dt} = i_L(1 - \frac{i_c - i_L}{\frac{T_s}{2L}v_s + MT_s}) - \frac{v_o}{R}. \tag{18b}$$

The large signal characteristics of the current programmed Buck-Boost converter can be obtained by either integrating (18) or entering the averaged circuit model (Fig. 7) into PSPICE.

C. Current Programmed Ćuk Converter

The effectiveness of the proposed averaged circuit model can be shown when it is applied to complicated topologies, such as a Ćuk converter. The averaged circuit model for Ćuk converter at duty ratio control is given in Fig. 3(c). In a Ćuk converter, the active switch is turned off when the current through it, $(i_{L1} + i_{L2})$, plus the artificial ramp, reaches the control signal i_c. Therefore, we have:

$$i_{L1} + i_{L2} = i_c - \frac{1}{2}m_1\alpha T_s - \alpha MT_s, \tag{19}$$

where m_1 is the rising slope of the switching current and can be expressed as follows:

$$m_1 = \frac{v_s}{L_1} + \frac{v_{C1} - v_o}{L_2}. \tag{20}$$

The duty ratio α can be found from (19) and (20) as follows:

$$\alpha = \frac{i_c - (i_{L1} + i_{L2})}{\frac{T_s}{2}(\frac{v_s}{L_1} + \frac{v_{C1} - v_o}{L_2}) + MT_s}. \tag{21}$$

Therefore, the averaged switch current i_Q and the averaged diode voltage v_d can be expressed as follows:

$$i_Q = \alpha(i_{L1} + i_{L2}) = (i_{L1} + i_{L2})\frac{i_c - (i_{L1} + i_{L2})}{\frac{T_s}{2}(\frac{v_s}{L_1} + \frac{v_{C1} - v_o}{L_2}) + MT_s}, \tag{22}$$

$$v_d = \alpha v_{C1} = v_{C1}\frac{i_c - (i_{L1} + i_{L2})}{\frac{T_s}{2}(\frac{v_s}{L_1} + \frac{v_{C1} - v_o}{L_2}) + MT_s}, \tag{23}$$

where v_{C1} is the voltage across capacitor C_1. Replacing the active switch Q with the controlled current source i_Q of (22), and replacing the diode with the controlled voltage source v_d of (23), the large signal averaged circuit model for current

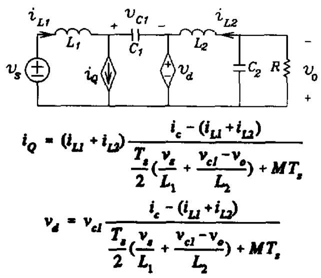


Fig. 8. Large signal averaged circuit model for current programmed Ćuk converter $i_Q = (i_{L1}+i_{L2})\frac{i_c-(i_{L1}+i_{L2})}{\frac{T_s}{2}(\frac{v_s}{L_1}+\frac{v_{C1}-v_o}{L_2})+MT_s}$, $v_d = v_{C1}\frac{i_c-(i_{L1}+i_{L2})}{\frac{T_s}{2}(\frac{v_s}{L_1}+\frac{v_{C1}-v_o}{L_2})+MT_s}$.

programmed Ćuk converter can be derived as shown in Fig. 8. The state space differential equations can also be obtained from the averaged circuit model of Fig. 8 straightforwardly, and the large signal characteristics can also be obtained accordingly.

D. Current Programmed Buck Converter with an Input Filter

The topology of the Buck converter with an input filter is shown in Fig. 9(a). When switch Q is on, the inductor current i_{L2} flows through it, and the capacitor voltage v_{C1} applies to the diode. The averaged circuit model under direct duty ratio control can be obtained as shown in Fig. 9(b). Under the current programmed control, the switch is turned off when the inductor current i_{L2} plus the artificial ramp reaches the control signal i_c. The rising slope, m_1, of the inductor current i_{L2} is now as follows:

$$m_1 = \frac{v_{C1} - v_o}{L_2}. \tag{24}$$

Therefore, the duty ratio α can be expressed as follows:

$$\alpha = \frac{i_c - i_{L2}}{\frac{T_s}{2L}(v_{C1} - v_o) + MT_s}. \tag{25}$$

Following the same procedure, the averaged switch current i_Q and diode voltage v_d can be found as follows:

$$i_Q = \alpha i_{L2} = i_{L2}\frac{i_c - i_{L2}}{\frac{T_s}{2L}(v_{C1} - v_o) + MT_s} \tag{26}$$

$$v_d = \alpha v_{C1} = v_{C1}\frac{i_c - i_{L2}}{\frac{T_s}{2L}(v_{C1} - v_o) + MT_s}. \tag{27}$$

The averaged circuit model is now obtained by replacing the switch Q and diode D with the controlled current i_Q (26), and controlled voltage v_d (27), respectively, as shown in Fig. 9(c). The state space differential equations can also be derived from this model.

In this and previous sections, the technique used to derive the large signal model for the current programmed converters is proposed, and the large signal models for the most commonly used switching converter topologies, Buck, Boost, Buck-Boost, and Ćuk, are derived. The same technique can also be used to derive the averaged circuit model for other switching converters under current programmed control. The proposed averaged circuit model keeps the same topology

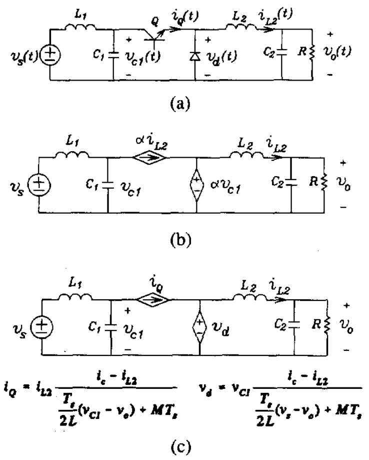


Fig. 9. Derivation of the averaged circuit model for current programmed Buck converter with an input filter. (a) Topology. (b) Averaged circuit model at duty ratio control $i_Q = i_{L2}\frac{i_c-i_{L2}}{\frac{T_s}{2L}(v_{C1}-v_o)+MT_s}$, $v_d = v_{C1}\frac{i_c-i_{L2}}{\frac{T_s}{2L}(v_s-v_o)+MT_s}$. (c) Averaged circuit model at current programmed control.

as the switching converter under investigation. The only difference between the model and the converter is that the active switch is replaced by a controlled current source, with its value equal to the averaged current flowing through it, and the diode is replaced by a controlled voltage source, with its value equal to the averaged voltage across it. The duty ratio is no longer present in the averaged circuit model.

From the above analysis, it is demonstrated that the mathematical manipulation to derive the large signal model is very simple and straightforward. The large signal characteristics of the current programmed converters can be analyzed by integrating the state space differential equations, or by a circuit simulation software package, because the averaged circuit model is readily compatible with such a package.

When the controlled current source i_Q and the controlled voltage source v_d are linearized around the operating point, a small signal model can be obtained, and, consequently, the small signal transfer functions can be derived. The steady state output voltage can also be found by setting all the circuit variables time-invariant and replacing the inductor and capacitor in the model with a short circuit and an open circuit, respectively. This is discussed in more detail in the next section, with the Buck converter as an example.

V. Characteristics of Current Programmed Buck Converter

In this section, the steady-state and small signal models of the current programmed Buck converter are derived from the large signal averaged circuit model proposed in Section III. Its steady state, small signal, and large signal characteristics are investigated in detail to illustrate the powerfulness of the proposed large signal model. In the following analysis, the lower-case letter denotes the averaged value, the upper-case letter denotes steady-state value and a hat $\hat{}$ above a symbol denotes the small signal perturbation.

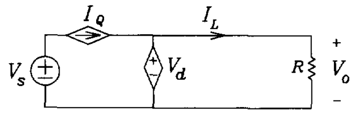


Fig. 10. Steady-state model of current programmed Buck converter.

A. Steady State

In steady state, the voltage across the inductor and the current through the capacitor are 0, and all the circuit variables are time-invariant. The steady-state averaged circuit can be derived, as shown in Fig. 10, where I_Q and v_d are as follows:

$$I_Q = I_L \frac{I_c - I_L}{\frac{T_s}{2L}(V_s - V_o) + MT_s}, \tag{28}$$

$$V_d = V_s \frac{I_c - I_L}{\frac{T_s}{2L}(V_s - V_o) + MT_s}. \tag{29}$$

Noticing that $V_d = V_o$, the steady-state output voltage can be solved easily as follows:

$$V_o = \frac{\frac{T_s V_s}{2L} + \frac{V_s}{R} + MT_s - \sqrt{\Delta}}{T_s/L}. \tag{30}$$

Another root is discarded because it is larger than the supply voltage and Δ is as follows:

$$\Delta = \left(\frac{T_s V_s}{2L} + \frac{V_s}{R} + MT_s\right)^2 - \frac{2T_s}{L} V_s I_c. \tag{31}$$

Equation (30) shows that in the current programmed control, the steady-state output voltage is no longer proportional to the supply voltage. The load resistor, filter inductor, switching period, and so forth, will also affect the output voltage.

According to the low entropy expression of quadratic equation proposed in [12], (30) can be rewritten as follows:

$$V_o = \frac{I_c R}{1 + \frac{1}{K} + \frac{MRT_s}{V_s}} \frac{1}{F}, \tag{32}$$

where K is the conduction parameter [4] defined as $K = 2L/(RT_s)$, and we have:

$$F = \frac{1}{2} + \frac{1}{2}\sqrt{1 - 4Q^2}, \tag{33}$$

in which

$$Q^2 = \frac{I_c R}{KV_s\left(1 + \frac{1}{K}\frac{MRT_s}{V_s}\right)^2}. \tag{34}$$

It can be observed from (32) that when Q is small, V_o is linear with I_c, because $F \approx 1$. Therefore, the initial part of V_o versus I_c curve is linear, as shown in Fig. 12(a). The circuit parameters are $V_s = 25$ V, $L = 230$ μH, $R = 5$ Ω, $T_s = 40$ μs, and $M = 0.075$ A/μs. The straight line with $F = 1$ is also plotted in Fig. 12(a). When I_c becomes large, V_o and I_c relation deviates from linearity, because Q increases; hence, $F < 1$.

Fig. 12(b) gives the relation between the output voltage and the supply voltage when the control signal I_c is kept unchanged at 5 A. This curve shows that when the supply

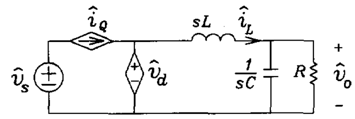


Fig. 11. Small signal model of current programmed Buck converter expressions for $\hat{i}_Q$ and $\hat{v}_d$ given in (32) and (33).

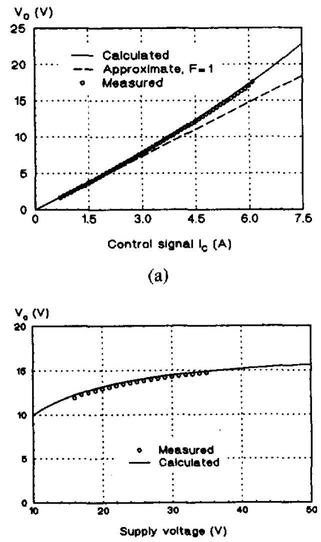


Fig. 12. Steady-state characteristics. (a) V_o vs. I_c ($V_s = 25$ V). (b) V_o vs. V_s ($I_c = 5$ A).

voltage changes, the output voltage does not change proportionally. This phenomenon can also be explained from (32). Because of the constant terms, 1 and $1/K$, the output voltage V_o is not so sensitive to the change of V_s. The supply voltage feedforward inherent in the current programmed control is clearly illustrated.

B. Small Signal Transfer Function

In the large signal model of the current programmed Buck converter (Fig. 5), only the controlled current source i_Q and controlled voltage source v_d are nonlinear components. All the other parts are linear. The i_Q and v_d can be linearized at small signal assumption. The small signal models for i_Q and v_d can be obtained by substituting all the circuit variables in (8) and (9), with steady-state value and its small signal perturbation, i.e., $x = X + \hat{x}$, and by neglecting the products of the small signal perturbation. The results can be expressed as follows:

$$\hat{i}_Q = \overline{\alpha}\hat{i}_L + a_c\hat{i}_c - a_L\hat{i}_L - a_s\hat{v}_s + a_o\hat{v}_o, \tag{35}$$

and

$$\hat{v}_d = \overline{\alpha}\hat{v}_s + b_c\hat{i}_c - b_L\hat{i}_L - b_s\hat{v}_s + b_o\hat{v}_o, \tag{36}$$

where $\overline{\alpha} = V_o/V_s$ is the steady-state voltage gain and:

$$a_L = a_c = \frac{I_L}{a}, \quad a_s = a_o = \frac{T_s}{2L}\frac{\overline{\alpha} I_L}{a},$$

$$b_L = b_c = \frac{a_L R}{\overline{\alpha}}, \quad b_s = b_o = \frac{a_o R}{\overline{\alpha}}, \tag{37}$$

$$a = \frac{T_s}{2L}(V_s - V_o) + MT_s. \tag{38}$$

Therefore, the small signal averaged circuit model can be obtained, and is shown in Fig. 11, where the expressions for $\hat{i}_Q$ and $\hat{v}_d$ are given in (32) and (33). The topology of the small signal averaged circuit model is the same as the Buck converter.

From the small signal averaged circuit model, the small signal transfer function can be derived easily as follows:

$$\hat{v}_o = \frac{(\overline{\alpha} - b_s)\hat{v}_s + b_c \hat{i}_c}{s^2 LC + \left(\frac{L}{R} + b_c C\right)s + 1 + \frac{b_L}{R} - b_o}. \tag{39}$$

Although the appearance of the above equation is different from what has been derived from the state space averaging method [4], they are actually the same, as shown in the Appendix.

From (39), it is noticed that under current programmed control, more damping is introduced as another term, $b_c Cs$, is added to the term, sL/R, so that the poles of the system become real, not complex, as in the case of direct duty ratio control. The supply voltage feedforward inherent in the current programmed Buck converter is also shown in (39) as the coefficient of $\hat{v}_s$ becomes $(\overline{\alpha} - b_s)$ for the current programmed control.

Other interesting small signal characteristics, such as the current loop gain, $\hat{i}_L/\hat{i}_c$, input impedance, $\hat{v}_s/\hat{i}_Q$, output impedance, $\hat{v}_o/\hat{i}_o$, can all be derived from the small signal model given in Fig. 11, and the details are not presented here.

From the above discussion, it is shown clearly that the steady-state output voltage and the small signal transfer functions can be derived from the large signal averaged circuit model conveniently and straightforwardly. Complicated mathematical manipulation is avoided. For different current controlled converters, the same procedure can be applied, and their steady-state output voltage and small signal transfer functions can be derived in the same way.

C. Large Signal Characteristics

Large signal characteristics of the current programmed Buck converter can be analyzed using the averaged circuit model shown in Fig. 5. The phase plane trajectory [9] is a good method by which to study the large signal characteristics of a second order nonlinear system, and is used here to analyze the current programmed Buck converter.

The circuit parameters for the analysis are $V_s = 25V$, $L = 230\ \mu$H, $C = 167\ \mu$F, $R = 5\ \Omega$, $R_L = 0.1\ \Omega$, $T_s = 40\ \mu$s, $M = 0.075$ A/μs. Here R_L is the sum of the sampling resistor and the winding resistor of the inductor. An evaluation version of PSPICE is used to calculate the large signal characteristics, because the model in Fig. 5 can be entered directly into PSPICE. Fig. 13 gives the phase plane trajectory for different initial conditions of the inductor and capacitor voltage when the control signal i_c is 5 A. The steady-state output voltage V_o is 13.8 V, and inductor current I_L is 2.76 A. The value of the x-axis and y-axis is the dynamic value of the output voltage and inductor current, i.e., the actual value minus the steady state value. It can be observed from the phase plane trajectory that the current controlled Buck converter has two separated real eigenvalues at the vicinity of the operating point. The change of the inductor current is much faster than that of the output voltage. This phenomenon is understandable, because in the current programmed control, it is the inductor current that we try to control.

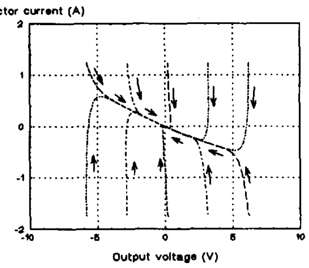


Fig. 13. Phase plane trajectory for different initial conditions.

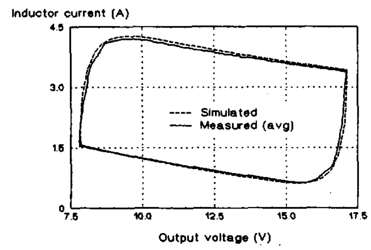


Fig. 14. Simulated and measured phase plane trajectory (I_c between 3 A and 6 A).

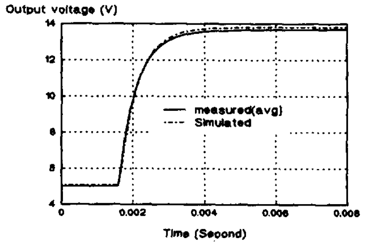
When the control signal steps between $i_c = 3$ A and $i_c = 6$ A, the phase plane trajectory of the system is shown in Fig. 14 (dashed line). It can be observed that when i_c steps from 3 A to 6 A, the inductor current rises rapidly at first, but the output voltage changes little. The inductor current overshoots a little bit, and then decays to the new steady-state value. A similar phenomenon happens when i_c steps from 6 A to 3 A.

For the time domain, when the control variable i_c steps from 2 A to 5 A, the response of the output voltage and the inductor current is given in Fig. 15 (dashed line). The time domain response of the output voltage is well damped, as shown in Fig. 15(a). The overshoot of the inductor current is observed clearly from Fig. 15(b).

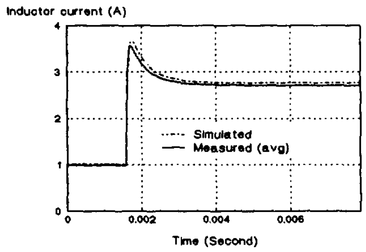
In this section, the characteristics of the current programmed Buck converter is analyzed thoroughly by using the large signal model proposed in the paper. The steady-state output voltage and the small signal transfer function are all derived from this large signal averaged circuit model. The large signal behavior is analyzed by both the phase plane trajectories and the time domain response when the control signal has a large signal step change. The analytical results will be verified by the experiments in next section.

VI. Experimental Verification

An experimental prototype of a current programmed Buck converter, with and without an input filter, are breadboarded

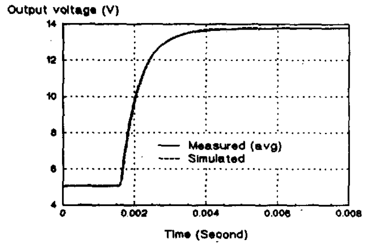


(a)

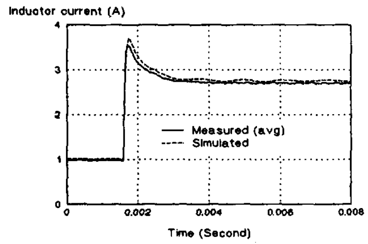


(b)

Fig. 15. Simulated and measured response of Buck converter without input filter. (a) Output voltage. (b) Inductor current.



(a)



(b)

Fig. 16. Simulated and measured response of Buck converter with input filter. (a) Output voltage. (b) Inductor current.

to verify the analysis. The circuit parameters are the same as before, i.e., $V_s = 25$ V, $L = 230$ μH, $C = 167$ μF, $R = 5$ Ω, $R_L = 0.1$ Ω, $M = 0.075$ A/μs. The input filter has the parameter $L_1 = 205$ μH, $C_1 = 163$ μF, and $R_{L1} = 0.123$ Ω.

Fig. 12 gives the steady-state characteristics of the current programmed Buck converter. Fig. 12(a) gives the measured and the calculated relation (from (30)) between the control signal I_c and output voltage V_o when the supply voltage is kept at 25 V. Fig. 12(b) gives the measured and calculated (from (30)) relation between V_s and V_o when $I_c = 5$ A. It is shown that the measured and the calculated output voltage is very close. The small difference is introduced by the sampling resistor of the inductor current and the winding resistor of the inductor, which are not considered in (30). It is demonstrated, therefore, that the steady state model derived from the averaged circuit model is valid to predict the steady-state behavior of the current programmed converters. Because the small signal transfer function is exactly the same as that derived at [4], which has been verified experimentally, the validity of the small signal model derived from the averaged circuit model is therefore verified.

Phase plane trajectory and time domain response are used to verify the large signal model. In the large signal dynamic tests, the Nicolet 310 digital scope is used to record the response of the inductor current and output voltage. The sample rate of the oscilloscope is set at 20 points per switching cycle, which is high enough to recover the actual waveform, according to the sample theorem. The measured data is transferred into a personal computer, taking the average value over one switching cycle, because it is the averaged value that is of interest, and then plotted at the same graph with the simulated results.

When the control signal steps between 3 A and 6 A, the output voltage and the inductor current step between 7.83 V and 17.13 V, and between 1.57 A and 3.43 A, respectively. The measured averaged phase plane trajectory is plotted in Fig. 14, as shown in the solid line. The simulated phase plane trajectory, as shown in the dashed line, is also plotted at the same graph. The measured phase plane trajectory is very close to the calculated one.

When the control signal i_c steps from 2 A to 5 A, the output voltage changes from 5 V to 13.7 V, and the inductor current changes from 1 A to 2.7 A. The transient response of the actual averaged output voltage and the averaged inductor current are shown in Fig. 15(a) and 15(b), respectively. The simulated response is also plotted in the same graph. The measured data are very close to the simulated data. This shows that when the control signal has a very large change, the model can predict the response of the current programmed Buck converter very well.

In order to illustrate that the technique proposed in the paper can handle complicated topology without any difficulty, an input filter is added to the Buck converter tested above. The response of the inductor current and the output voltage are also measured when the control signal has the same large signal step, i.e., from 2 A to 5 A, as shown in Fig. 16(a) and 16(b), respectively. It is shown that simulated response is also very close to the measured response.

The experimental results presented in this section demonstrated that the proposed averaged circuit model for current programmed converter can predict the steady-state and large signal dynamic characteristics accurately. Its validity for small signal dynamic response has already been proved, because the small signal transfer function is the same as that derived in [4], which has been verified experimentally.

VII. Conclusion

A technique used to derive the general unified large signal model for the current programmed converter is proposed in this paper. The steady-state relation and small signal transfer functions of the current programmed converters can all be derived from this large signal model.

The large signal averaged circuit model of the current programmed DC-to-DC switching converter is obtained by:

1) replacing the active switch with a controlled current source that is dependent on the current command, i_c, supply voltage and some other circuit variables, and that has the same value as the averaged current flowing through the active switch;
2) replacing the diode with a controlled voltage source that is also dependent on the current command, i_c, and some other circuit variables, and that has the same value as the averaged voltage across the diode; and
3) keeping all the other parts of the switching converter unchanged.

The procedure to derive the averaged circuit model is straightforward and convenient. The model is simple, because it has the same topology as the switching converter under investigation. The large signal characteristics of the current programmed converters can be obtained easily, either by the nonlinear state space differential equations or by a circuit simulation software package. The effect of the input filter, output filter, and parasitic parameters can all be included in the model and can be evaluated without any difficulty.

The large signal averaged circuit models for current programmed Buck, Boost, Buck-boost converters, and more complicated Ćuk converter and Buck converter with an input filter are proposed in the paper. The steady-state relation and small signal transfer function of the Buck converter under current programmed control are derived from its large signal averaged circuit model. Its large signal characteristics are investigated by both the phase plane trajectory and time domain response. The experimental prototypes are built to verify the averaged circuit model. The measurements and the theoretical predictions are very close.

The large signal averaged circuit model presented in this paper gives a global view of the system dynamics, which enables the designer to understand the limits of the system and helps the designer to achieve robust control. Its compatibility with the circuit simulation software package makes it very easy to analyze the large signal, as well as the small signal, characteristics of the current programmed converters, even when various parasitic parameters are included. A current programmed switching power supply with global stability can be obtained with the help of the proposed averaged circuit model.

APPENDIX

In this Appendix, it is shown that the small signal transfer function derived from the large signal averaged circuit model proposed in the paper is the same as that derived from the y-parameter model from Middlebrook's paper [4].

The small signal transfer function derived from the large signal averaged circuit model is given in (36) and is rewritten as follows:

$$\hat{v}_o = \frac{(\overline{\alpha} - b_s)\hat{v}_s + b_c\hat{i}_c}{s^2LC + (\frac{L}{R} + b_cC)s + 1 + \frac{b_L}{R} - b_o}. \quad \text{(A1)}$$

The expressions for b_s, b_L, b_c, and b_o are given in (34). Substituting (34) into (A1), the coefficients of the numerator can be simplified as follows:

$$\overline{\alpha} - b_s = \frac{V_o}{aV_s}\left(MT_s - \frac{T_s}{2L}V_o\right), \quad \text{(A2)}$$

$$b_c = \frac{V_s}{a}, \quad \text{(A3)}$$

where the expression for a can be found in (35). Therefore, the small signal transfer function can be expressed as follows:

$$\hat{v}_o = \frac{\frac{V_o}{V_s}(MT_s - \frac{T_s}{2L}V_o)\hat{v}_s + V_s\hat{i}_c}{k_0s^2 + k_1s + k_2}, \quad \text{(A4)}$$

where

$$k_0 = aLC, \quad k_1 = a\left(\frac{L}{R} + b_cC\right), \quad k_2 = a\left(1 + \frac{b_L}{R} - b_o\right). \quad \text{(A5)}$$

Substituting (34) and (35) into (A5), the expression of k_0, k_1, and k_2 can be simplified as follows:

$$\begin{aligned} k_0 &= \frac{CT_s}{2}(V_s - V_o) + LCMT_s \\ k_1 &= \frac{T_s}{2R}(V_s - V_o) + \frac{L}{R}MT_s + CV_s \\ k_2 &= V_s(\frac{T_s}{2L} + \frac{1}{R}) + MT_s - \frac{T_s}{L}V_o. \end{aligned} \quad \text{(A6)}$$

The y-parameter model [4] is given in Fig. 17, and the expressions of the coefficients are as follows:

$$\begin{aligned} y_{21} &= -\frac{D(nD' - 1)}{KR}\frac{1}{1 + s/\omega_c}, \quad y_{2c} = -\frac{1}{1 + s/\omega_c}, \\ y_{22} &= \frac{(nD' - D)}{KR}\frac{1}{1 + s/\omega_c}, \end{aligned} \quad \text{(A7)}$$

where

$$\begin{aligned} n &= 1 + \frac{2M}{M_1}, \quad \omega_c = \frac{\omega_s}{\pi nD'}, \quad \omega_s = 2\pi/T_s, \quad K = \frac{2L}{RT_s}, \\ D &= \frac{V_o}{V_s}, \quad D' = 1 - D, \end{aligned} \quad \text{(A8)}$$

where M is the slope of the artificial ramp, and M_1 is the steady-state slope of the rising segment of the inductor current. For Buck converter, we have:

$$M_1 = \frac{V_s - V_o}{L}. \quad \text{(A9)}$$

The small signal output voltage can be expressed as follows:

$$\hat{v}_o = \frac{-y_{21}\hat{v}_s - y_{2c}\hat{i}_c}{y_{22} + sC + \frac{1}{R}} = \frac{N_s\hat{v}_s + V_s\hat{i}_c}{a_0s^2 + a_1s + a_2}, \quad \text{(A10)}$$

where

$$\begin{aligned} N_s &= \frac{D(nD' - 1)}{KR}V_s, \quad a_0 = \frac{V_sC}{\omega_c}, \\ a_1 &= V_s(C + \frac{1}{R\omega_c}), \quad a_2 = \frac{V_s}{R}(1 + \frac{nD' - D}{K}). \end{aligned} \quad \text{(A11)}$$

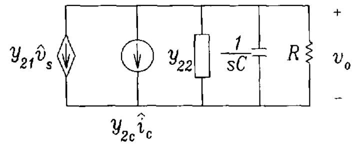


Fig. 17. y-parameter model for current programmed Buck converter.

Substituting (A8) into (A11), the coefficients N_s, a_0, a_1, and a_2 can be simplified as follows:

$$N_s = \frac{V_o}{V_s}(MT_s - \frac{T_s}{2L}V_o), \tag{A12}$$

and

$$\begin{aligned} a_0 &= \frac{CT_s}{2}(V_s - V_o) + LCMT_s \\ a_1 &= \frac{T_s}{2R}(V_s - V_o) + \frac{L}{R}MT_s + CV_s \\ a_2 &= V_s\left(\frac{T_s}{2L} + \frac{1}{R}\right) + MT_s - \frac{T_s}{L}V_o. \end{aligned} \tag{A13}$$

Comparing (A10) with (A4) and taking into account (A6), (A12), and (A13), it is obvious that the small signal transfer function derived from the large signal averaged circuit model proposed in the paper is the same as that derived from the y-parameter model.

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