A Novel Method to Achieve Zero-Voltage Regulation in Buck Converter

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***Abstract*— The Function Control law for Buck converter is derived to achieve zero voltage regulation of the output voltage. A new method to retrieve the low frequency component of the inductor voltage is proposed and analyzed. The stability of the closed loop system using proportional and differential controller is analyzed. The effect of the supply voltage and load current disturbance is also studied. The analysis, computer simulation by PSPICE and experimental results illustrate that excellent performance can be achieved by the Function Control law.**

I. Introduction

FOR a switching power supply with constant output voltage, it is desirable that the output voltage remains unchanged in both steady state and even at transient operation under the disturbances from either the supply voltage or load current. In other words, zero-voltage regulation of the output voltage of the switching converter is achieved when the output voltage is independent of the supply voltage and the load current. Therefore, neither the supply voltage nor the load current will affect the output voltage at both steady state and dynamic operation.

The most commonly used control method is the direct duty ratio control [1]–[4], in which the output voltage is fedback and compared with the reference voltage and the output of the error amplifier is used to control the duty ratio. Unfortunately, the supply voltage and load current disturbance cannot be eliminated by this control law because only after the output voltage changes can the correcting action begin.

Another commonly used control method is the current programming control [5]–[8], where the inductor current is also fedback together with the output voltage. The output of the error amplifier serves as the command for the inductor current which determines when the switch is turned off. By proper design, the current-programmed control can eliminate the supply voltage disturbance. Unfortunately, it can not eliminate the load current disturbance.

A lot of effort have been made to eliminate the effect of the load current disturbance and supply voltage disturbance [9], [10], [14], [15]. In [14], a switching power supply with optimum dynamic performance is breadboarded using twenty four operational amplifiers, eight analog function modules, four voltage comparators, an analog multiplexer and three logic gates. It suffers from offset and drift problems and it needs too much hardware to be a practical design.

In [15], a near-optimum switching power supply is obtained using feedforward of output current and input voltage. There are two potential problems with this technique. One is that this technique is based on the hysteretic current control and the switching frequency will change which is not desirable. The other is that it is difficult to sense the output current and additional loss is usually incurred.

In [9], the derivative of the load current is fedback and by tuning the differentiating time constant and DC gain constant, zero output impedance is achieved. However, it is very difficult to differentiate the load current in the switching converter because of the switching noise. In addition, the output voltage is very sensitive to the parameters of the power stage, i.e., the value of the filter inductor and its parasitic resistor should be tuned carefully to meet a critical condition. The effect of the supply voltage disturbance cannot be eliminated by this technique.

On the other hand, in [10], the supply voltage is fed forward and the switch is turned off when the integral of the supply voltage reaches a reference voltage. The disturbance of the supply voltage is thus eliminated. Unfortunately, the technique can do nothing about the load current disturbance.

In order to achieve zero voltage regulation of the output voltage, the authors have proposed a Function Control law in a conference [11] and encouraging results have been obtained by the preliminary analysis. Unfortunately, the way to retrieve some circuit variable is crude, which limits the stable operation of the system only at slow change of the supply voltage and load current. The system becomes unstable when there is a step change of supply voltage or load current. Only the proportional controller is used in that paper and the stability of the closed loop system was not analyzed.

In this paper, the Function Control law for the Buck converter is derived and the physical meaning of the zero voltage regulation is explained. A new method to retrieve the low frequency component of the inductor voltage, which is essential to implement the Function Control law, is proposed and analyzed. The large signal closed loop characteristics are also analyzed to ensure the stable operation of the system and to study the effect of the supply voltage and load current disturbances. The analysis, computer simulation by PSPICE and experimental results show that excellent performance can be achieved by Function Control. A Buck converter with almost zero voltage regulation of the output voltage is obtained. A Buck converter with the conventional direct

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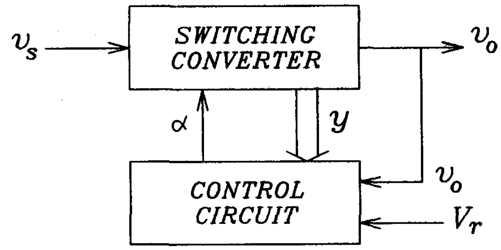


Fig. 1. General block diagram for the switching converter.

duty ratio control under optimal design is also simulated and breadboarded as a comparison to show the significant improvement achieved by Function Control.

II. Function Control Law for Buck Converter

Generally speaking, a switching power supply consists of two basic parts, one is the power stage, or the switching converter, and the other is the control circuit, as shown in Fig. 1, where V_r is the reference voltage, y denotes the combination of the feedbacks, α is the duty ratio. The power stage controls the power absorbed from the unregulated supply voltage (v_s) and provides a regulated constant output voltage (v_o) at the load. The main purpose of the control circuit is to generate a proper duty ratio according to the circuit condition in order to reduce the variation of the output voltage as much as possible when the supply voltage or load current changes. For different control laws, such as duty ratio control, current programmed control, etc., the effect of the supply voltage or load current disturbance is different.

In order to achieve zero-voltage regulation, i.e., to eliminate the effect of the supply voltage or load current disturbance, the control strategy should be particularly constructed and the feedbacks should be properly selected so that the control circuit can provide the exact duty ratio required by the power stage. The closed loop output voltage is thus independent of either the supply voltage or the load current and is determined only by the reference voltage.

The above objective can be realized when the principle of the Function Control law is used to formulate the control strategy of the Buck converter. Fig. 2(a) gives the topology of the Buck converter and Fig. 2(b) gives its large signal low frequency averaged equivalent circuit model which can be derived from the technique of equivalent controlled source model [12]. By this equivalent model, the active switch is modeled by a controlled current source with its value equal to the averaging current flowing through the switch over one switching cycle, i.e., $i_s = \alpha i$, for Buck converter, where i is the averaged inductor current. The diode is modeled by a controlled voltage source with its value equal to the averaging voltage across it over one switching cycle, i.e., $v_d = \alpha v_s$ for Buck converter.

From Fig. 2(b), the output voltage can be expressed as

$$v_o = \alpha_p v_s - v_L \tag{1}$$

where v_L is the averaged value of the inductor voltage and α_p is the duty ratio required for the switching converter. This

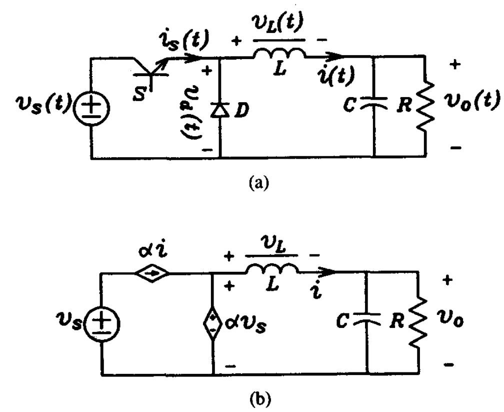


Fig. 2. (a) Buck converter topology, and (b) its low frequency average model.

α_p can be expressed as

$$\alpha_p = \frac{v_o + v_L}{v_s}. \tag{2}$$

Eq. (2) defines the duty ratio required by the Buck converter at a specific operating point of v_o, v_L and v_s.

The control circuit can now be constructed to generate the duty ratio. Let the input and output relation of the control circuit be formulated as

$$\alpha_c = \frac{K(V_r - v_o) + v_L}{v_s} \tag{3}$$

where V_r is the reference voltage, K is the gain of the proportional error amplifier. Duty ratio α_c denotes the duty ratio generated by the control circuit.

In the practical circuit, the output of the control circuit is connected to the gate of the active switch in the power stage, making $\alpha_p = \alpha_c$. Therefore, the closed loop characteristic can be obtained by equating (2) and (3) as

$$\frac{v_o + v_L}{v_s} = \frac{K(V_r - v_o) + v_L}{v_s}. \tag{4}$$

From (4), the output voltage can be found as

$$v_o = \frac{K}{k+1} V_r. \tag{5}$$

It is noted that the output voltage v_o at the left side of (5) is not a steady state value, but rather an averaged one, which includes both the steady state value V_0 and the dynamic one, $\tilde{v}_o$, i.e., $v_o = V_0 + \tilde{v}_o$. The reference voltage V_r is a constant for constant output voltage application. Eq. (5), thus, shows that by the control law (3), the closed loop averaged output voltage is forced to be proportional to a constant reference voltage. This result means that the closed loop output voltage is independent of the supply voltage and the load current. In other words, the averaged output voltage remains unchanged even when there is disturbance from either the supply voltage or the load current. Zero-voltage regulation of the output voltage is, therefore, achieved.

The control law (3) is nonlinear. The duty ratio is proportional to the sum of the averaged inductor voltage and the

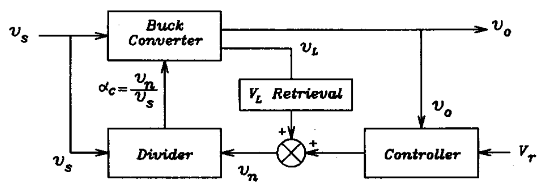


Fig. 3. Block diagram of function control.

output of the error amplifier and is inversely proportional to the supply voltage. Nonlinear control law combined with the inherent nonlinear Buck converter has resulted, in this case, in a linear closed loop system. The averaged closed loop output voltage is proportional to the reference voltage.

When the supply voltage changes, the duty ratio will react immediately and change accordingly to such an extent that it can cancel the effect of the supply voltage disturbance. Thus the output voltage keeps unchanged.

The feedback of the averaged inductor voltage is used to correct the disturbance of the load current. The inductor voltage is proportional to the rate of change of the inductor current, which is the sum of the load current and the capacitor current as shown in Fig. 2(b), i.e., $v_L = Ldi/dt$ and $i = i_c + i_o$. As soon as there is a tendency for the load current to change, i.e., even before the load current has actually changed, the correction action of the control circuit begins. The generated duty ratio is accurate enough so that the low frequency averaged output voltage does not change.

The block diagram of the Function Controlled Buck converter is illustrated in Fig. 3. The low frequency component of the inductor voltage is retrieved and is added with the output of the controller to formulate the numerator of the control law (3). Although only the proportional controller is considered in (3), the proportional-integral, proportional-differential, etc., controller can also be used. A divider is needed to generate the required duty ratio α_c so that the duty ratio of the control circuit output is equal to the ratio of v_n and v_s (Fig. 3). In practical circuit, this divider can be implemented simply by an Opamp and a comparator.

It is worthwhile to point out here that in the above analysis, only the averaged value is considered. No small signal assumption is made. Therefore, the above analysis is also valid for large signal variation. The following sections will discuss the implementation of this control law and the stability of the closed loop system.

III. The Method to Retrieve the Low Frequency Component of Inductor Voltage

From the discussion of the previous section, the low frequency component of the inductor voltage is needed to formulate the Function Control law. In this section, it is shown that the inductor voltage can be expressed as the sum of a low frequency component and a switching frequency component with zero averaging value over one switching period. A circuit that can retrieve this low frequency component and suppress greatly the switching frequency component is proposed and analyzed.

At the continuous conduction mode, the instantaneous inductor voltage of the Buck converter is

$$v_L(t) = k(t)v_s - v_o \tag{6}$$

where $k(t)$ is the switching function and is defined as

$$k(t) = \begin{cases} 1 & \text{when } S \text{ is on} \\ 0 & \text{when } S \text{ is off.} \end{cases} \tag{7}$$

The switching function $k(t)$ consists of the low frequency component α and the multiples of the switching frequency, i.e.,

$$k(t) = \alpha + k_1 \sin(\omega_s t + \varphi_1) + \sum_{n=2}^{\infty} k_n \sin(n\omega_s t + \varphi_n) \tag{8}$$

where α is the duty ratio of the switch and $\omega_s = 2\pi f_s$, f_s is the switching frequency and

$$k_n = \frac{2}{\pi n}|\sin n\alpha\pi|, \quad \varphi_n = \frac{\pi}{2} - \pi\alpha n, \quad n = 1, 2, \ldots \tag{9}$$

Because the corner frequency of the output filter is much lower than the switching frequency for a well-designed converter, the frequency components higher than the switching frequency can be neglected and the inductor voltage can be expressed as

$$v_L(t) = \alpha v_s - v_o + k_1 v_s \sin\left(\omega_s t + \frac{\pi}{2} - \varphi_1\right). \tag{10}$$

Because the supply voltage v_s does not change within one switching cycle at low frequency assumption, the above equation can be rewritten as

$$v_L(t) = \alpha v_s - v_o + A \sin\left(\omega_s t + \frac{\pi}{2} - \varphi_1\right) \tag{11}$$

where $A = k_1 \times v_s$ does not change over one switching cycle.

Equation (11) means that the inductor voltage $v_L(t)$ consists of two components, one is the low frequency component which is changing in amplitude and the other is the switching frequency component with constant amplitude and zero averaging value over one switching period.

From the discussion of the previous section, the low frequency component of the inductor voltage is needed to implement the Function Control law and to achieve zero voltage regulation of the output voltage. The high frequency component of the inductor voltage should be somehow eliminated.

The most straightforward way is to use a low pass filter. However, this is not a good choice because in practical circuit the amplitude of the switching frequency component of $v_L(t)$ is much higher than that of low frequency component.

The low frequency component of the inductor voltage can be retrieved by the circuit shown in Fig. 4. Suppose the input of the circuit, v_1, consists of two components, one is the high frequency component, v_{sw}, and the other is the low frequency signal component, v_{sig}, i.e.,

$$v_1 = A_1 \sin\omega t + A_2 \sin(\omega_s t + \varphi) = v_{sig} + v_{sw} \tag{12}$$

where ω is signal frequency and ω_s is switching frequency. The input voltage, v_1, is at first converted into a proportional current $i = v_1/R$ by an opamp. The current i is then used to charge the capacitor C. The output voltage, v_2, is the sampled value of the capacitor voltage at every switching cycle

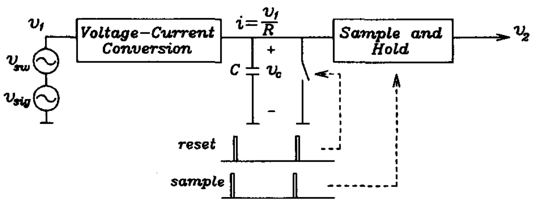


Fig. 4. Circuit to retrieve the low frequency component of the inductor voltage.

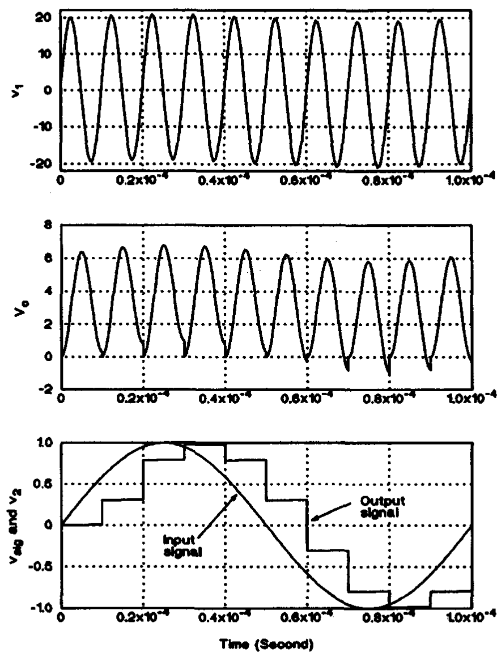


Fig. 5. Typical waveforms of Fig. 4.

$T_s = 2\pi/\omega_s$. The capacitor is reset after sampling so that no previous information is kept. Because the average value of the high frequency component, v_{sw}, over the sampling period is zero, the output v_2 contains only the information of the low frequency term of the input signal, v_{sig}. Fig. 5 gives the typical waveforms of the circuit in Fig. 4, where the amplitude of the high frequency component $A_2 = 20$ V and the amplitude of the signal component $A_1 = 1$ V, only 5% of A_2. The switching frequency is 100 kHz and the signal frequency is 10 kHz. Fig. 5 shows that the input signal can be retrieved from the high frequency noise. It is observed that the fundamental component of the output voltage v_2, delays the input low frequency signal, v_{sig}, one switching period. The switching frequency component of the output voltage is very small.

Although this observation is made only for the special case, it holds true for the general cases. PSPICE simulation is made to obtain the input output relation of the circuit in Fig. 4. In the simulation, the amplitude of the low frequency and switching frequency components are $A_1 = 1$, and $A_2 = 20$, respectively, which means that the high frequency component is much higher than the useful signal as the actual case. The switching

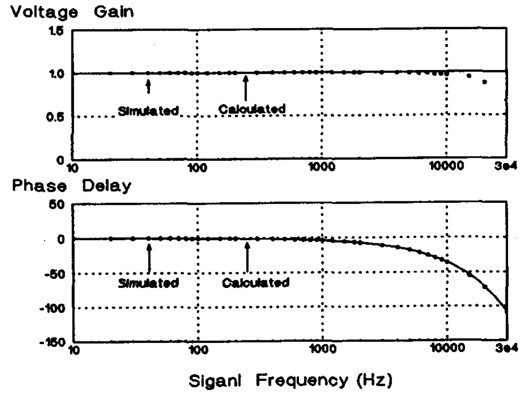


Fig. 6. Simulated frequency response of the circuit in Fig. 4.

frequency is chosen as 100 kHz. When the frequency of the signal, v_{sig}, varies, the actual response of the circuit shown in Fig. 4 is simulated and the amplitude and the phase delay of the output voltage is calculated accordingly. The frequency response of the circuit, v_2/v_{sig}, is thus obtained and is plotted in Fig. 6, as shown by the cycles.

Because of sampling and resetting, the high frequency component is greatly attenuated. For example, at signal frequency 10 kHz, the lowest high frequency components are at 90 kHz and 110 kHz. The amplitude of the high frequency components is: $V_{90\,\mathrm{kHz}} = 0.1074$ (V) and $V_{110\,\mathrm{kHz}} = 0.08677$ (V). In other words, at the output of the circuit of Fig. 4, the largest amplitude of high frequency component is only 0.1074/20 = 0.00537 = −45.4 (dB) of the input one. On the other hand, the amplitude of the useful signal is 0.9658 (V), with phase delay of only −36°. Therefore, the circuit in Fig. 4 can attenuate the high frequency component greatly and retrieve the useful signal.

In Fig. 6, the frequency response of the pure time delay $H(s) = e^{-sT_s}$ at $T_s = 10\ \mu$S is also plotted as shown in solid line. It is evident that the low frequency input to output relation for the circuit of Fig. 4 can be expressed approximately as

$$H(s) = \frac{V_2(s)}{V_{sig}(s)} = e^{-sT_s}. \tag{13}$$

In the time domain, Eq. (13) means that the output voltage v_2 delays the signal voltage v_{sig} by one switching cycle T_s, i.e.,

$$v_2(t) = v_{sig}(t - T_s). \tag{14}$$

When the frequency components higher than the switching frequency are considered, the above conclusion does not change because the average value of these components over one switching period is zero.

The circuit shown in Fig. 4 will be used to retrieve the low frequency component of the inductor voltage so as to implement the Function Control law.

IV. Stability Analysis

In this section, the stability of the Buck converter by Function Control is analyzed. The effect of the supply voltage and load current disturbance is also studied. The analysis is

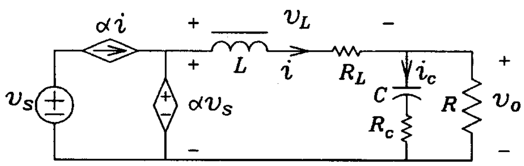


Fig. 7. Equivalent circuit of Buck converter.

based on the low frequency large signal averaged equivalent circuit model of the Buck converter, as shown in Fig. 7. The winding resistor of the inductor, R_L, and the equivalent series resistor (ESR) of the filter capacitor, R_C, are also included.

As discussed in the previous section, the retrieved low frequency component of inductor voltage delays the actual one by one switching period, T_s, i.e.,

$$v_L' = v_L(t - T_s) \tag{15}$$

where v_L' denotes the retrieved inductor voltage. The Function Control law is constructed as

$$\alpha_c = \frac{v_L' + K(V_r - v_o) - K_d \frac{dv_o}{dt}}{v_s}. \tag{16}$$

Here the proportional and differential controller is used as this is what has been implemented. In (16), K_d is the time constant of the differentiator. Instead of differentiating the output voltage directly, $\frac{dv_o}{dt}$ can be obtained easily for the Buck converter by sensing the current in the filter capacitor so that the effect of the noise in v_o will be eliminated. From (16) and the relation of the power stage, (2), the closed loop output voltage can be expressed as

$$v_L + v_o = v_L(t - T_s) + K(V_r - v_o) - K_d \frac{dv_o}{dt}. \tag{17}$$

The closed loop steady state output voltage can be found as

$$V_0 = \frac{K}{K+1} V_r. \tag{18}$$

The averaged output voltage v_o and inductor voltage v_L can be expressed as: $v_o = V_0 + \tilde{v}_o$ and $v_L = V_L + \tilde{v}_L$, where the upper case letters denote the steady state value and a tilde "$\sim$" above a variable denotes a dynamic component. It should be noticed that no small signal assumption is made. The dynamic equation can also be found from (17) as

$$\tilde{v}_L + \tilde{v}_o = \tilde{v}_L(t - T_s) - K\tilde{v}_o - K_d \frac{d\tilde{v}_o}{dt}. \tag{19}$$

Eq. (19) is a linear differential equation with time delay. Its Laplace transformation can be found as

$$V_L(s) + V_0(s) = e^{-sT_s} V_L(s) - KV_0(s) - sK_d V_0(s) \tag{20}$$

where all the initial conditions are set to zero.

Because of the low frequency assumption, e^{-sT_s} can be expressed approximately as

$$e^{-sT_s} = 1 - sT_s \tag{21}$$

and the relation between $V_0(s)$ and $V_L(s)$ can be found from Fig. 7 as

$$V_L(s) = \left(\frac{s^2 LC}{1 + sR_cC} + \frac{sL}{R} + \frac{sR_LC}{1 + sR_cC} + \frac{R_L}{R}\right) V_0(s). \tag{22}$$

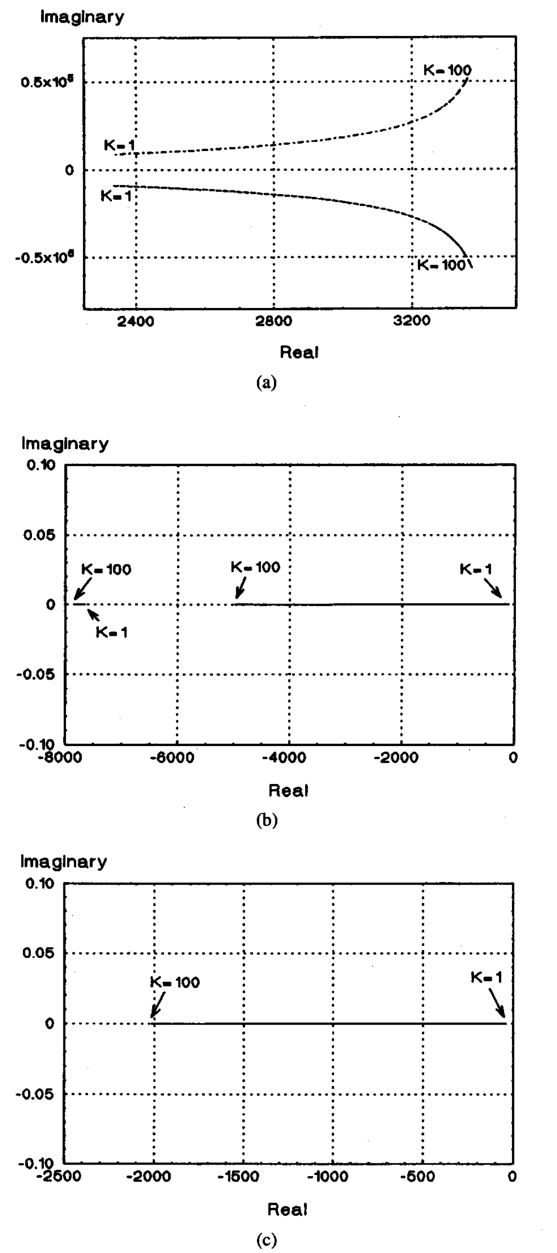


Fig. 8. Root locus of (23) for different K_d; (a) $K_d = 0$; (b) $K_d = 0.02$; (c) $K_d = 0.05$.

Substituting (21) and (22) into (20), the closed loop dynamic output voltage can be derived as

$$(b_0 s^3 + b_1 s^2 + b_2 s + b_3) V_0(s) = 0 \tag{23}$$

where the coefficients are

$$b_0 = T_s LC\left(1 + \frac{R_c}{R}\right)$$

$$b_1 = \frac{T_s L}{R} + T_s R_c C + \frac{T_s R_L R_c C}{R} + K_d R_c C \tag{24}$$

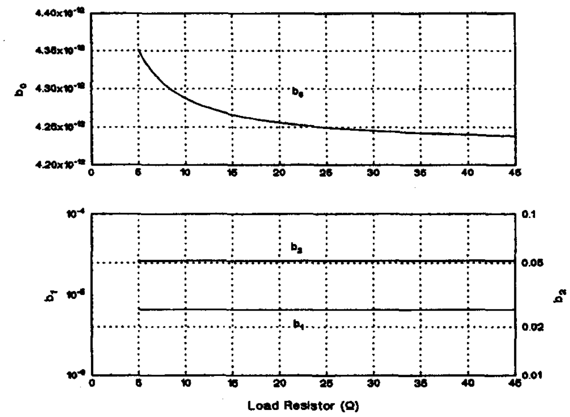


Fig. 9. Effect of the load resistor R_L on coefficients of (34).

$$b_2 = K_d + \frac{T_s R_L}{R} + (K+1)R_c C, \quad b_3 = K + 1.$$

The effect of K and K_d on the stability of the system is investigated and the root-locus of (23) is plotted, when K changes from 1 to 100, as shown in Fig. 8 for $K_d = 0$, 0.02, 0.05. The parameters of the power stage used in the simulation are: $T_s = 20\mu S$, $L = 240$ μH, $C = 880$ μF, $R = 12$ Ω and $R_L = 0.05$ Ω, $R_C = 0.15$ Ω, here T_s is the switching period.

Fig. 8(a) gives two conjugate segments of the root-locus when $K_d = 0$, which is equivalent to the proportional controller. Another segment lies at the left half plane and is not plotted here. Obviously the system is not stable as two roots lie in the right half s-plane. When the gain of the dv_o/dt feedback, K_d, is 0.02, all the roots of (23) become real and lie in the left half s-plane. The system is stable. Fig. 8(b) gives the two segments of the root locus. The third one lies far away left of these two and is not plotted here. Further increasing K_d, Fig. 8(c) gives one segment of root-locus when $K_d = 0.05$. In this case, the other segment of the root locus shrinks to almost a point at -7610. The third one still lies at the far left of these two. The system's behavior is dominated by the low frequency eigenvalue. Comparing the root-locus at different K_d, it is shown that the system by the Function Control law (16) can be stabilized when the gain K_d is proper.

Now let's study the effect of the disturbance of the supply voltage and the load resistor (load current). As the supply voltage v_s does not appear in the closed loop characteristic equation of the system, (23), the output voltage is thus independent of the supply voltage.

The effect of the load resistance, R, is studied. First, it is noted from (24) that if the switching period T_s is set to zero, which means that there is no delay in retrieving the low frequency component of the inductor voltage, the coefficients, b_0, b_1, b_2 and b_3, will be independent of the value of the load resistor R. Therefore, because of the time delay in retrieving the inductor voltage, the effect of the load resistor can not be eliminated completely, but its effect is very insignificant as shown in the following analysis.

When the load resistor R changes from 5 Ω to 45 Ω for $K = 10$ and $K_d = 0.05$, the corresponding change of the coefficients b_0, b_1, b_2 are plotted, as shown in Fig. 9 and b_3 is independent of the load resistance. First, it is noted that b_0 (in the order of 4×10^{-12}) is much smaller than b_1 (at around 6.6×10^{-6}). The first term at the left side of the system characteristic equation, (23), becomes significant only when the frequency of interest is higher than 10 kHz, as shown in Fig. 10. In other words, the other three terms of (23) will dominate the characteristics of the closed loop system for low frequency range. Second, when the load resistor R has a very large variation, the relative variation of b_0, b_1 and b_2 is: $\frac{\Delta b_0}{b_0} = 2.6\%$, $\frac{\Delta b_1}{b_1} = 0.013\%$ and $\frac{\Delta b_2}{b_2} = 0.0004\%$. The variation of b_1 and b_2 is extremely small. This result, together with the fact that b_0 has little effect on the system, indicates that the closed loop output voltage is insensitive to the load resistor change. This implication will also be verified by the computer simulation and the experiments in the following sections.

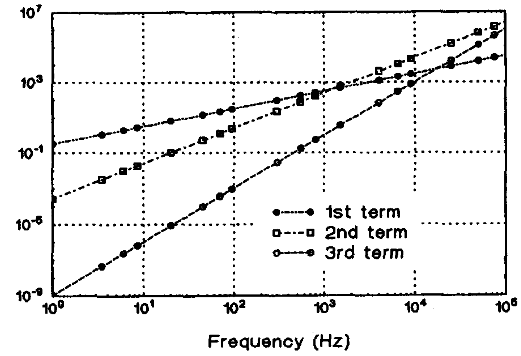


Fig. 10. Values of first three terms in (34) versus frequency.

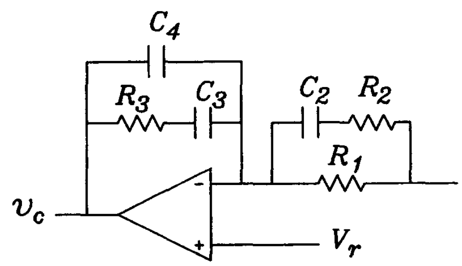


Fig. 11. Optimal controller for direct duty ratio control $R_1 = 2.7$ KΩ, $R_2 = 8.2$ KΩ, $R_3 = 2.4$ KΩ, $C_2 = 0.05$ μF, $C_3 = 0.15$ μF, $C_4 = 2.4$ nF.

With proportional controller, i.e., $K_d = 0$, the system is unstable as discussed above. When proportional-integral (PI) controller is used, the stability of the closed loop system is also analyzed and is not stable. The details are not presented here.

The above analysis shows that with the Function Control law, (16), the root locus of the system can be put in the left half s-plane to ensure the stable operation of the system. The effect of the supply voltage can be eliminated completely. The effect of the load resistor on the output voltage is very small and insignificant.

It should also be noticed that in the above analysis, no small signal assumption is made. With the nonlinear Function Control law, the nonlinear Buck converter is changed into a linear system. Therefore, the analysis is greatly simplified. All

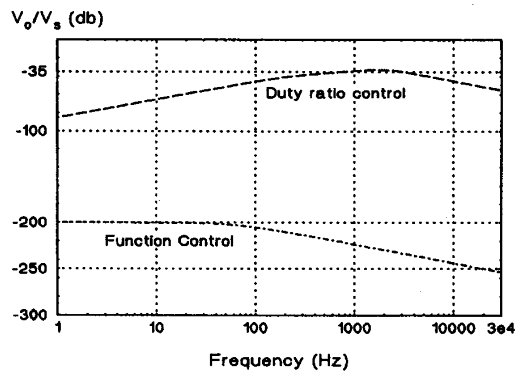


Fig. 12. Audio susceptibility for Function Control and direct duty ratio control.

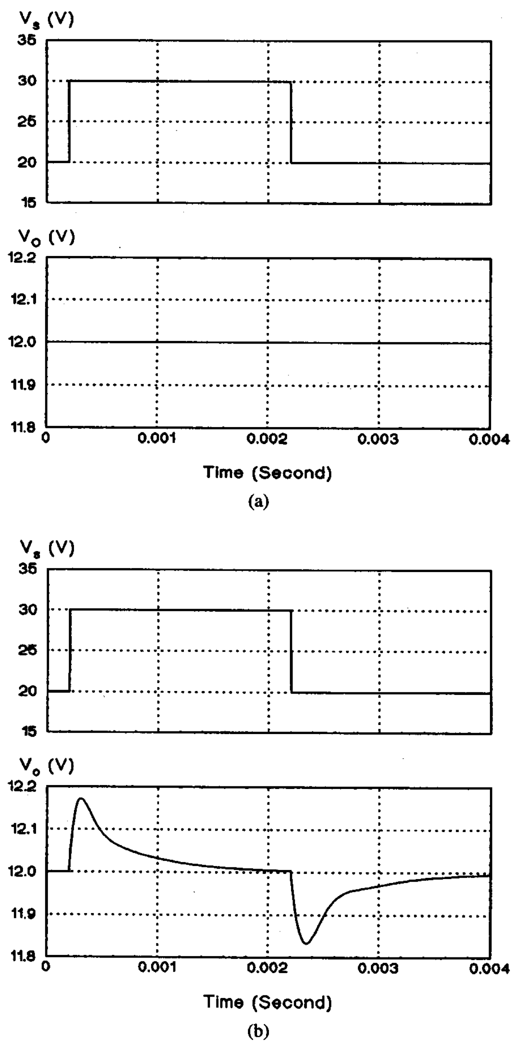


Fig. 13. Effect of the supply voltage step: (a) Function Control; (b) direct duty ratio control.

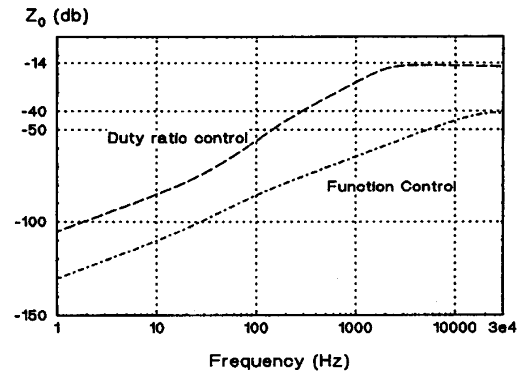


Fig. 14. Output impedance for Function Control and direct duty ratio control ($1\ \Omega = 1$ dB).

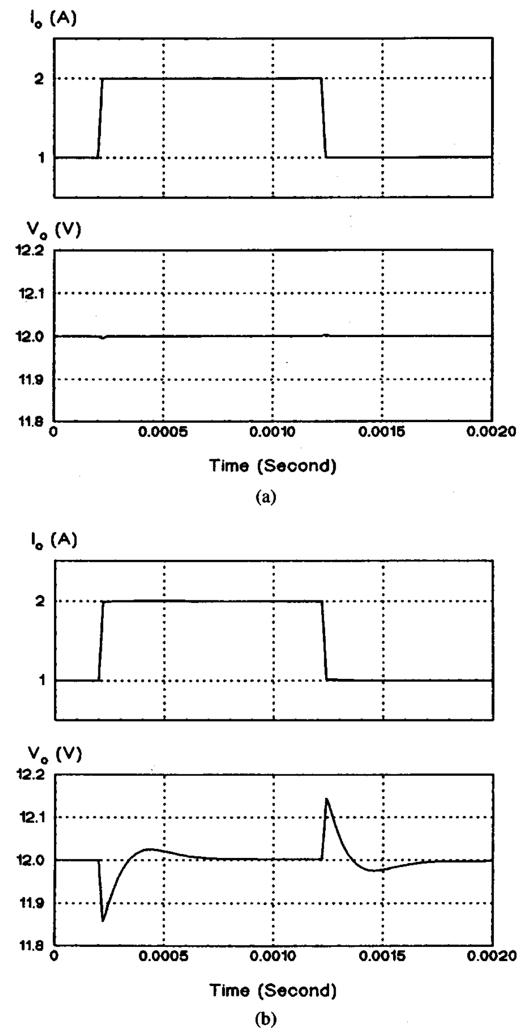


Fig. 15. Effect of the load current step: (a) Function Control; (b) direct duty ratio control.

the linear system theory can be used to analyze the dynamic behavior of the closed loop system and the results are valid for both the small signal and large signal variation.

V. Computer Simulation

In this section, the dynamic characteristics of the Function Controlled Buck switching regulator are simulated by PSPICE.

The dynamic characteristics of the conventional direct duty ratio control under the optimal design are also simulated as a comparison.

A. Model Description

The equivalent circuit of Fig. 7 is used for the PSPICE simulation. The time delay of the retrieved inductor voltage is modeled by the Analog Behavioral Modeling of the PSPICE by setting $E_{VL} = e^{-sT_s}V_L$. The parameters of the power circuit are the same as used in the previous section and $K = 10$, $K_d = 0.05$.

In order to demonstrate the significant improvements achieved by the Function Control, the dynamic behavior of a Buck converter with the conventional direct duty ratio control is also simulated. The parameters of the power stage are the same as above. The controller is implemented by an optimal controller with a three-pole and two-zero compensator proposed by [13]. Fig. 11 shows the compensator. The first pole is placed at the origin for tight DC regulation. The second pole is placed in order to cancel the equivalent series resistance (ESR) zero. The final pole is placed at approximately one half of the switching frequency (here $f_s = 50$ kHz) to reduce the switching noise. The first zero is placed slightly before the resonant frequency of the output filter to avoid a conditional stable system and the second zero is placed after the resonant frequency to obtain the adequate phase boost. The integrator gain is chosen to obtain 50° of phase margin. The selected parameters according to the above rule are also listed in Fig. 11.

B. Simulation Results

The effect of the supply voltage is at first simulated. Fig. 12 gives the audio susceptibility of the Buck converter by Function Control and by direct duty ratio control. The worst audio susceptibility for Function Control is −200 (dB) while for the optimal conventional control, the worst audio susceptibility is −35 (dB). It can be observed from Fig. 12 that the audio susceptibility of Function Controlled Buck converter is always significantly lower than the conventional one.

When the supply voltage has a large step change, from 20 V to 30 V, the output voltage does not change at all for the Function Controlled Buck converter, as shown in Fig. 13(a). This shows that the disturbance of the supply voltage is eliminated by the Function Control. For the direct duty ratio control, on the other hand, the output voltage has a large deviation and long transient time under the same supply voltage disturbance, as shown in Fig. 13(b). The significant performance improvement by Function Control is obviously observed.

The effect of the load disturbance is also studied. Fig. 14 gives the closed loop output impedance of the Buck converter by the Function Control and by the direct duty ratio control. The largest output impedance for Function Controlled Buck converter is −40 (dB) while for the direct duty ratio control, the largest output impedance is −14 (dB). The output impedance is much lower in the case of the Function Control for the entire frequency range.

When the load current change between 1 A and 2 A, the response of the output voltage of the Function Controlled Buck converter is given in Fig. 15(a). The peak deviation is less than 5 mV. For the direct duty ratio control, the change of the output voltage is 140 mV for the same current disturbance and takes longer time to recover, as shown in Fig. 15(b). Therefore, the effect of the load resistor disturbance can be greatly reduced by the Function Control.

It is shown from the above simulation that significant improvement can be achieved by the Function Control law. The Buck converter by the Function Control law can eliminate the disturbance of the supply voltage. The effect of the load resistor disturbance is extremely small. An almost zero voltage regulation switching power supply is obtained.

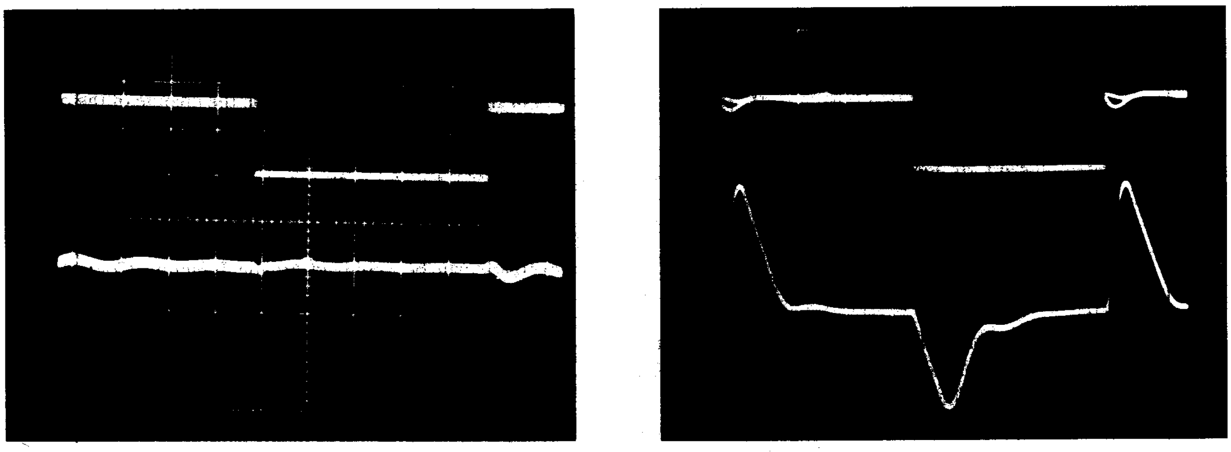
VI. Experimental Results

The Function Controlled Buck converter is breadboarded. The circuit is connected according to the block diagram shown in Fig. 3. The circuit shown in Fig. 4 is used to retrieve the low frequency component of the inductor voltage. The controller is a proportional and differential controller, where the $\frac{dv_o}{dt}$ is retrieved by sensing the current flowing through the filter capacitor. The parameters of the power stage and the controller are the same as those used in the analysis. The output voltage is 12 V. The gain of inductor voltage feedback is adjusted to as close as possible to unity. A Buck converter with same power stage parameters but under the conventional direct duty ratio control employing the optimal compensator, as shown in Fig. 11, is also breadboarded as a comparison. Figs. 16 and 17 gives the experimental results.

When the supply voltage has a large signal step change between 20 V and 28 V (with the load resistor 12 Ω), Fig. 16(a) shows the response of the output voltage of the Buck converter by Function Control. Insignificantly dynamic change is observed for step change of the supply voltage. For the conventional duty ratio control, the response of the output voltage is given in Fig. 16(b). Significant change and longer recovery time are observed. The significant improvement of the input voltage suppression by the Function Control is illustrated clearly.

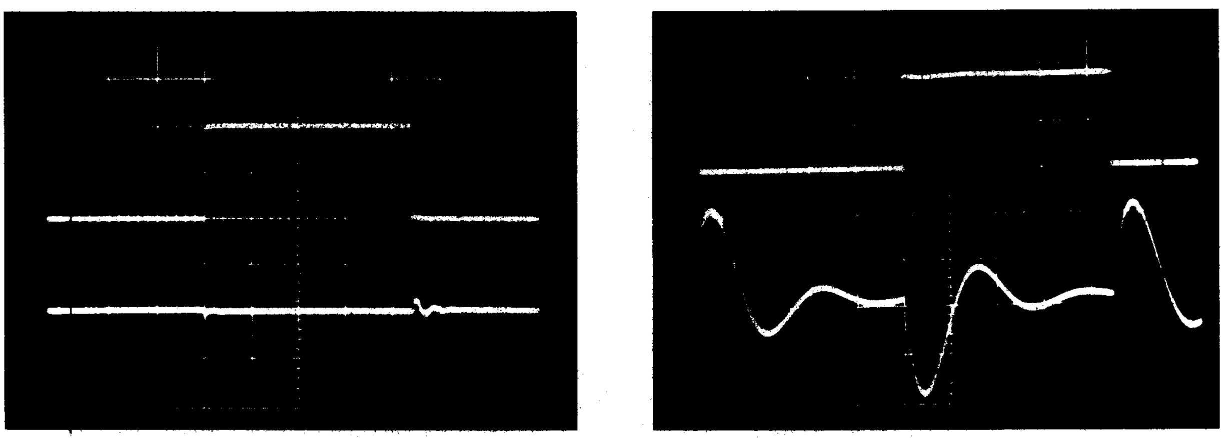
Fig. 17 shows the effect of the load resistor disturbance on the output voltage when the supply voltage is 25 V. When the load current has a large signal step change between 1 A and 2 A (load resistor steps between 12 Ω and 6 Ω), the response of the output voltage by Function Control is given in Fig. 17(a). The oscillogram shows that the change of the output voltage is very insignificant even at the transient period for the load resistor change. For the conventional duty ratio control, the response of the output voltage under the same load resistor disturbance is given in Fig. 17(b). Large peak deviation and longer recovery time are observed. It is demonstrated, therefore, that great performance improvement can be achieved by the Function Control law.

From the experimental results presented in this section, it can be seen that by the Function Control law, the closed loop output voltage is very insensitive to the supply voltage and



(a) (b)

Fig. 16. Effect of the supply voltage step: (a) Function Control, upper: v_s, 5 V/div, lower: v_o, 50 mV/div, time: 1 mS/div; (b) direct duty ratio control, upper: v_s, 5 V/div, lower: v_o, 0.5 V/div, time: 2 mS/div.



(a) (b)

Fig. 17. Effect of the load current step: (a) Function Control, upper: i_o, 0.5 A/div, lower: v_o, 100 mV/div, time: 1 mS/div; (b) direct duty ratio control, upper: i_o, 0.5 A/div, lower: v_o, 100 mV/div, time: 1 mS/div.

load resistor disturbances even during the dynamic period. Zero voltage regulation of the output voltage is, thus, obtained.

VII. Conclusion

The study presented in this paper shows that a switching power supply with zero-voltage regulation of the output voltage can be obtained by the Function Control law. A method to retrieve the low frequency component of the inductor voltage and at the same time suppress greatly its high frequency components is proposed and analyzed in order to implement the Function Control law. The results indicate that this method is very effective in retrieving the low frequency component of the inductor voltage, which is crucial to achieve zero voltage regulation of the output voltage.

The nonlinear Function Control law converts the nonlinear Buck converter into a linear closed loop system. Therefore, the closed loop dynamic analysis is greatly simplified. The large signal characteristics of the closed loop system is analyzed and the impacts of different control parameters are studied. The stability of the closed loop system is ensured by the proper selection of the controller parameters based on the analysis presented in the paper. The analysis shows that the disturbance of the supply voltage can be eliminated completely and the closed loop output voltage is insensitive to the load resistor.

The Function Controlled Buck converter is simulated by PSPICE and a prototype is breadboarded. A Buck converter with the same power stage parameters but under direct duty ratio control with an optimal compensator is also simulated and breadboarded as a comparison. Both the computer simulation and the experimental results show that great improvement can be achieved by the Function Control law and an almost zero voltage regulation converter is obtained.

References

[1] N. Mohan, T. M. Undeland and W. P. Robbins, *Power Electronics: Converters, Applications and Design*. New York: John Wiley, 1989.
[2] M. H. Rashid, *Power Electronics: Circuits, Devices and Applications*. Englewood Cliffs, NJ: Prentice Hall, 1988.
[3] P. C. Sen, *Thyristor DC Drives*. New York: John Wiley, 1981.
[4] ——, *Principle of Electric Machines and Power Electronics*. New York: John Wiley, 1988.
[5] C. W. Deisch, "Simple switching control method changes power converter into a current source," *IEEE PESC Record*, 1978, pp. 300–306.
[6] A. Capel, G. Ferrante, D. O'Sullivan and A. Weinberg, "Application of injected current model for the dynamic analysis of switching regulators with new concept of LC^3 modulator," *IEEE PESC Record*, 1978, pp. 135–147.
[7] R. Redl and N. O. Sokal, "Current-mode control, five different types with the three basic classes of power converters: Small-signal AC and large-signal DC characterization, stability requirements and implementation of practical circuits," *IEEE PESC Record*, 1985, pp. 771–785.
[8] R. D. Middlebrook, "Topics in multi-loop regulators and current mode programming," *IEEE PESC Record*, 1985, pp. 716–732.
[9] L. D. Varga and N. A. Losic, "Synthesis of zero-impedance converter," *IEEE Trans. Power Electron.*, vol. 7, no. 1, pp. 152–170, Jan. 1992.
[10] K. M Smedley and S. Cuk, "One cycle control of switching converters," *IEEE PESC Record*, 1991, pp. 888–896.
[11] Y. F. Liu, P. C. Sen and S. P. Huang, "Function control—A novel strategy to achieve improved performance of the DC-to-DC switching regulators," *Record of IEEE Industrial Application Society Annual Meeting*, 1991, pp. 971–976.
[12] X. Z. Zhang and S. P. Huang, "Equivalent controlled source model for the PWM switching converters," *Electrical Journal*, no. 3, 1986 (in Chinese).
[13] D. M. Sable, R. B. Ridley and B. H. Cho, "Comparison of performance of single-loop and current-injection control for PWM converters that operate in both continuous and discontinuous modes of operation," *IEEE Trans. Power Electron.*, vol. 7, no. 1, pp. 136–142, Jan. 1992.
[14] W. W. Burns, III and T. G. Wilson, "Analytic derivation and evaluation of a state-trajectory control law for dc-to-dc converters," *IEEE PESC 1977 Record*, pp. 70–85.
[15] R. Redl and N. O. Sokal, "Near-optimum dynamic regulation of DC-DC converters using feed-forward of output current and input voltage with current-mode control," *IEEE Trans. Power Electron.*, vol. 1, no. 3, pp. 181–192, July 1986.

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