A Large Signal Dynamic Model for DC-to-DC Converters with Average Current Control

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***Abstract*—This paper presents a generalized model for average current control that can be applied to DC-to-DC converters such as the Buck, Boost, Buck-Boost and Ćuk converters. The proposed model consists of two parts: 1) an averaged DC-to-DC converter topology with all the switching elements replaced by dependent sources 2) an average current control scheme with a Pulse Width Modulation (PWM) model, which determines the duty cycles. This model can accurately predict the steady-state, small signal and large signal dynamic behavior for average current controlled DC-to-DC single switch converters. To verify the proposed model, simulated results were compared to experimental waveforms of an average current controlled Boost converter. The experimental results demonstrate that the model can correctly predict the steady-state, small signal and large signal dynamic behavior of the average current controlled Boost converter.**

Keywords-Large signal Mode; Average current control; DC-to-DC converters

I. Introduction

Average current control is commonly used for DC-to-DC converters and single-phase power factor correction (PFC) circuits. It has several advantages over peak current control such as the elimination of the external compensation ramp, increased gain for DC and low frequencies and improved immunity to noise in the sensed current signal [2], [3]. Simulation software packages such as Spice include integrated circuits modules that perform average current control. However, the problem with these types of circuit simulations is their long computation time, which is caused by several reasons:

1. Due to the complexity of the control algorithms and switching elements, long computational times will be required to generate the simulation waveforms. Particularly when the converter topologies and control circuitry are not linear [7].
2. Long computation time occurs when the time constant of the converter is much greater than the switching period. In order to show how these types of converters behave at steady-state, the operating time of the simulated circuit will be quite long.
3. Numerous switches in a topology will also increase the computation time. In such cases, there are many diodes and MOSFETs switching at high frequencies and multiple control circuitries. As a result, the overall computation time for obtaining simulation waveforms at steady-state can drastically increase due to the increased number of switching elements and overall complexity.

In order to reduce the long computation times, the complex converter circuit can be replaced with a simple equivalent model.

In the past, several technical papers have addressed average current control modeling for DC-to-DC converters [3-5]. The models presented in [4,5] only deals with the analysis of the small signal characteristics. Though [3] does address some large signal issues, it cannot be applied to converters with nonlinear transfer functions such as the Boost, Buck-Boost and Ćuk converters. With the method presented in [3], for nonlinear converters, only the small signal model can be obtained by using state-space averaging which involves perturbing and linearizing around an operation point.

In this paper, a new method of modeling average current control is proposed. This method takes advantage of advanced simulation software by utilizing the averaged circuit model rather than an equation form [1]. The model consists of two parts:

1. DC-to-DC single switch converter topology with all the switching elements replaced by dependent sources [1].
2. An average current control circuit with a PWM circuit model which determines the duty cycle value.

Using this model, the steady-state, small signal and large signal characteristics of the switching converter under average current control can be analyzed. This model uses the averaged converter method to model the DC-to-DC single switch converter [1]. It utilizes simulation tools such as Spice to evaluate the steady-state, small signal and large signal characteristics. The main advantages of the proposed model are listed as follows:

1. This model can predict the steady-state, small and large signal characteristics for all DC-to-DC converters using average current control.
2. Simplified circuit model is used instead of complex transfer function blocks.
3. The proposed model can determine the frequency response of all continuous conduction mode (CCM) operating conditions for all DC-DC converters.
4. By using the model, both small and large signal response can be generated, although the transfer functions can be mathematically derived from the model.
5. The proposed model is intuitive since the topology of the model is very similar to the topology of the circuit.

In this paper, the averaged model will be derived for the Boost topology in section II. In section III, the average current control transfer function for duty cycle is formulated. Section IV will deal with the DC-to-DC converter model, which consisted of the averaged model of the Boost converter combined with the average current control scheme. Section V will demonstrate how the proposed modeling technique can be applied to converters such as the Buck, Buck-Boost and Ćuk converters. In section VI, the validity of the model is verified through experimental analysis of an averaged current controlled Boost converter. The experimental waveforms were compared to the model simulations for large signal and small signal response. Section VII is the conclusion.

II. The Averaged Model

As mentioned in section I, replacing the circuit with a simpler converter model can reduce the computation time. Simplification of the model is gained by replacing the MOSFET and diode with non-switching dependent sources to produce an averaged non-switching converter model [1]. The MOSFET is modeled by a controlled current source, whose value is i_Q, the averaged value of the current through the MOSFET during each switching period. The diode is modeled by a controlled voltage source, whose value is v_G, the average voltage across the diode for each switching period. Fig. 1(b) illustrates the Boost converter with an added current sensing resistor. As long as the sensing resistor is small, its effect on Boost converter behavior is negligible. Fig. 1(a) shows the waveforms of the current through the MOSFET $i_Q(t)$ and the diode voltage $v_D(t)$ for one switching period. Low ripple assumption and ideal switches were assumed. The averaged Boost converter model can be derived from these waveforms [1]. The averaged model for a Boost converter with a sensing resistor is given in Fig. 1(c).

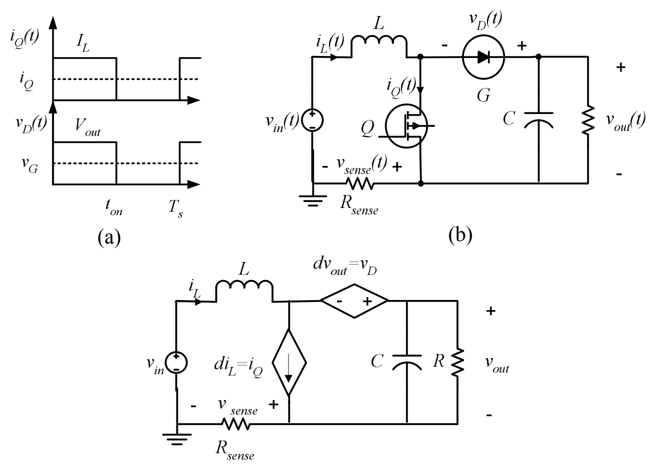


Fig. 1. (a) Switching and averaged waveforms of the Boost converter: the dashed line represents the averaged value of the waveform (b) Boost converter with R_{sense} (c) Averaged model for a Boost converter with R_{sense},

Fig. 1(b) shows that the averaged values of the current through the MOSFET and the voltage across the diode are related to the duty cycle. The duty cycle can be determined by (1), where t_{on} is the time the MOSFET is switched on and T_s is the switching period.

$$Duty = \frac{t_{on}}{T_s} = d \tag{1}$$

The controlled current source, which replaces the MOSFET in the averaged model, has a current value that is expressed in (2). The voltage value across the controlled voltage source that replaces the diode in the averaged model is expressed in (3). The current i_L is the current through the inductor when the MOSFET is on and v_{out} is the output voltage of the Boost converter. The values of the resistor, capacitor, inductor, and supply voltage remains unchanged in the model as compared to those of the Boost circuit because they are present for both on and off states of the MOSFET [1].

$$i_Q = \frac{t_{on}}{T_s} I_L = dI_L \tag{2}$$

$$v_D = \frac{t_{on}}{T_s} V_{out} = dV_{out} \tag{3}$$

For the averaged Boost converter model, it is shown that all the model waveforms can be determined by three variables: 1) the inductor current I_L when the MOSFET is on, 2) the output voltage of the converter V_{out} and 3) the duty cycle value d [1]. The averaged models for other types of converters such as Buck, Buck-Boost and the Ćuk converters can be derived using the same method [1].

III. Model of Average Current Controller

Average current control method is often used for applications where tight regulation of the current is needed, such as in PFC [8]. In average current control, the control variable is the reference current i_{ref} and it is compared to the sensed current signal. The sensed current signal is expressed in (4).

$$v_{sense} = R_{sense} \times i_L \tag{4}$$

With the sensed current and the reference current value, it is possible to determine the duty cycle value for average current control. This duty cycle value can then be fed into an averaged model of the DC-to-DC converter to obtain the desired waveforms. Fig. 2 illustrates the average current control scheme with a pulse width modulator.

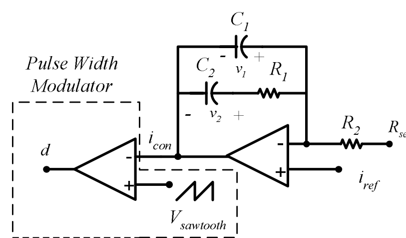


Fig. 2. Average current control scheme with a pulse width modulator

Since the feedback compensation for the op-amp is a low-pass network, the switching ripple in the inductor current i_L and v_{sense} can be neglected [8]. The sensed current signal v_{sense} is fed into R_2, which is connected to the inverting terminal of the op-amp. The non-inverting terminal of the op-amp is connected to the i_{ref}. The difference between v_{sense} and i_{ref} will be amplified by the compensation network to produce the signal i_{con}. i_{con} is then fed into the inverting input of the comparator to generate the duty cycle d. The non-inverting input of the comparator is a sawtooth signal with magnitude $V_{sawtooth}$ and frequency $f_{sawtooth}$. The output of the comparator d is the pulse width modulated (PWM) signal, which controls the duty cycle of the converter. The PWM circuit can be modeled as a constant multiplier $1/V_{sawtooth}$ and a voltage limiter which limits the duty cycle between d_{min} and d_{max}. Fig. 3 shows the PWM circuit model with average current control scheme.

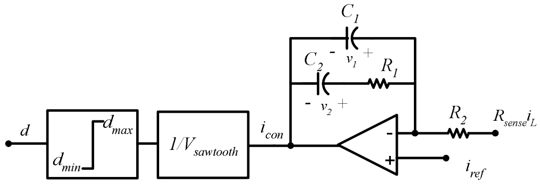


Fig. 3. Average current control scheme with the PWM model

From Fig. 3, the transfer function of duty cycle for average current control scheme can be derived as:

$$d(s) = \frac{1}{V_{sawtooth}}\left(I_{ref}(s) + H(s)(I_{ref}(s) - R_{sense} I_L(s))\right) \qquad (5)$$

where $H(s)$ is the transfer function from the inverting terminal to v_d

$$H(s) = \frac{K_c(1 + s/\omega_2)}{s(1 + s/\omega_1)}$$

and K_c, ω_1 and ω_2 are defined by

$$K_c = \frac{1}{R_2(C_1 + C_2)}, \quad \omega_1 = \frac{C_1 + C_2}{R_1 C_1 C_2}, \quad \omega_2 = \frac{1}{R_1 C_2}$$

This transfer function for duty is identical to what is derived in [3], [8]. The pole of $H(S)$ at the origin is used to increase DC gain of the current loop, the zero extends the crossover frequency and the high frequency pole is used to increase noise immunity [3], [4].

IV. The Boost Converter Model With Average Current Control

Several method of modeling average current mode control is proposed in [3-5]. However, the problem with those proposed models for small and large signal analysis is their complexity. The derived small signal transfer functions are at least 4[th] order, depending on the DC-to-DC converter. In addition, for large signal analysis, the derived differential equations can be quite complex.

Fig. 4(b) shows the complete model for a DC-to-DC Boost converter with average current control. It consists of two parts: the averaged model of the Boost converter and the average current control compensator with the PWM circuit model. For the average current mode control scheme illustrated in Fig. 2 and describe by (5), the relationship between the input (i_{ref}, v_{sense}) and the output (i_{con}) is linear. As a result, using the proposed method, the current loop can easily be modeled since the average current control compensator remains unchanged in the circuit (Fig. 4a) as compared to that of the model (Fig. 4b).

A. Steady-State analysis

When the model is in steady-state, the average magnitude of v_{sense} will be equal to the magnitude of the current reference i_{ref}. Therefore from (4), the relationship between the inductor current and the current reference can be expressed by

$$I_L = \frac{I_{ref}}{R_{sense}} \qquad (6)$$

It can be observed from (6) that by using average current control, the magnitude and the shape of i_{ref} will determine the magnitude and shape of the input current i_L.

The output voltage of the Boost converter can be expressed in terms of the reference current and the input voltage. By equating the input power to the output power of the Boost converter, the following equation can be derived.

$$V_{in} I_L = \frac{V_{out}^2}{R} + I_L^2 R_{sense} \qquad (7)$$

The relationship between the output voltage, input voltage and the current reference signal can be derived by substituting (6) into (7).

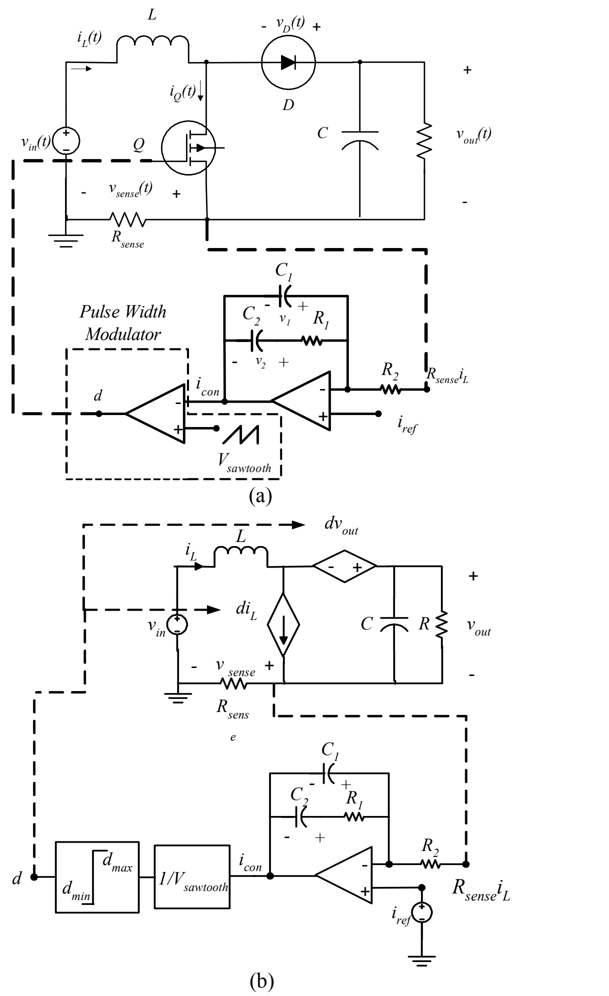


Fig. 4. (a) Averaged DC-to-DC Boost converter with average current control and PWM circuit (b) The proposed model: averaged DC-to-DC Boost converter model with average current control and PWM circuit model

$$\sqrt{\frac{RI_{ref}}{R_{sense}}\left[V_{in} - I_{ref}\right]} = V_{out} \qquad (8)$$

It can be observed from (8) that the output voltage is dependent on the load resister, the sensing resister, input voltage and the current reference signal. Equation 8 shows that the steady-state relationship between the input voltage and the output voltage and the relationship between the reference current and the output voltage is nonlinear. The nonlinearity is due to the square root term that defines the output voltage.

B. Small Signal Model and Transfer Functions

The proposed model takes advantage of advanced simulation software by utilizing the circuit representation of the model rather than an equation form to generate the small signal response. However, the small signal transfer function can still be mathematically derived from the proposed model.

Due to the nonlinear nature of the Boost converter, the model must be linearized using small signal assumption in order to obtain the small signal transfer functions. Each state variable x will consist of a steady-state value X and a small signal perturbation $\hat{x}$. The small signal transfer functions can be mathematically derived from the model by choosing an operating point, substituting a steady-state and small signal

$x = X + \hat{x}$ (where $\hat{x} << X$) into (5). Subsequently, the steady-state and higher order terms are excluded in order to obtain the small signal transfer functions. Fig. 5 show the model converted to the Laplace domain with small signal assumption. From Fig. 5, the transfer function for the output voltage and input current in terms of a small disturbance in the current reference $\hat{i}_{ref}$ can be determined.

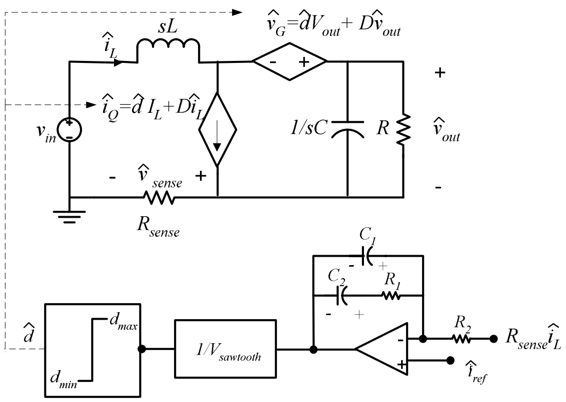


Fig. 5. The small signal model

Equations (9) and (10) are the transfer functions of the voltage and current dependent sources in the average converter model. $\hat{v}_G$ and $\hat{i}_Q$ were determined by linearizing (2), (3) and (5) at small signal assumption.

$$\hat{v}_g = \hat{d}V_{out} \times D\hat{v}_{out} = A_1\hat{v}_{out} + A_2\hat{i}_{ref} - A_3\hat{i}_L \qquad (9)$$

$$\hat{i}_Q = \hat{d}I_L \times D\hat{i}_L = A_4\hat{i}_L + A_5\hat{i}_{ref} \qquad (10)$$

where

$$A_1 = \frac{I_{ref} + KI_{ref} - KR_{sense}I_L}{V_{sawtooth}}, \quad A_2 = \frac{V_{out}(1+K)}{V_{sawtooth}}, \quad A_3 = \frac{V_{out}R_{sense}}{V_{sawtooth}},$$

$$A_4 = \frac{I_{ref} + KI_{ref} - 2KR_{sense}I_L}{V_{sawtooth}}, \quad A_5 = \frac{I_L(1+K)}{V_{sawtooth}}, \quad K = \frac{1 + sR_1C_2}{s^2C_1C_2R_1 + s(C_1 + C_2)}$$

From (9), (10) and the small signal model in Fig. 5, the transfer functions of $\hat{i}_L$ and $\hat{v}_{out}$ in terms of a small disturbance $\hat{i}_{ref}$ can be derived.

$$\hat{i}_L = \frac{\hat{v}_{in} + [A_2(sCR+1) + RA_5 - RA_1A_5]\hat{i}_{ref}}{(sL + A_3 + R_{sense})(sCR+1) - RA_1 + RA_1A_4 + R - RA_4} \qquad (11)$$

$$\hat{v}_{out} = \frac{[T_2 - 1]\hat{v}_{in} + [A_2T_2 - T_1(sL + A_3 + R_{sense})]\hat{i}_{ref}}{T_2[1 - A_1]} \qquad (12)$$

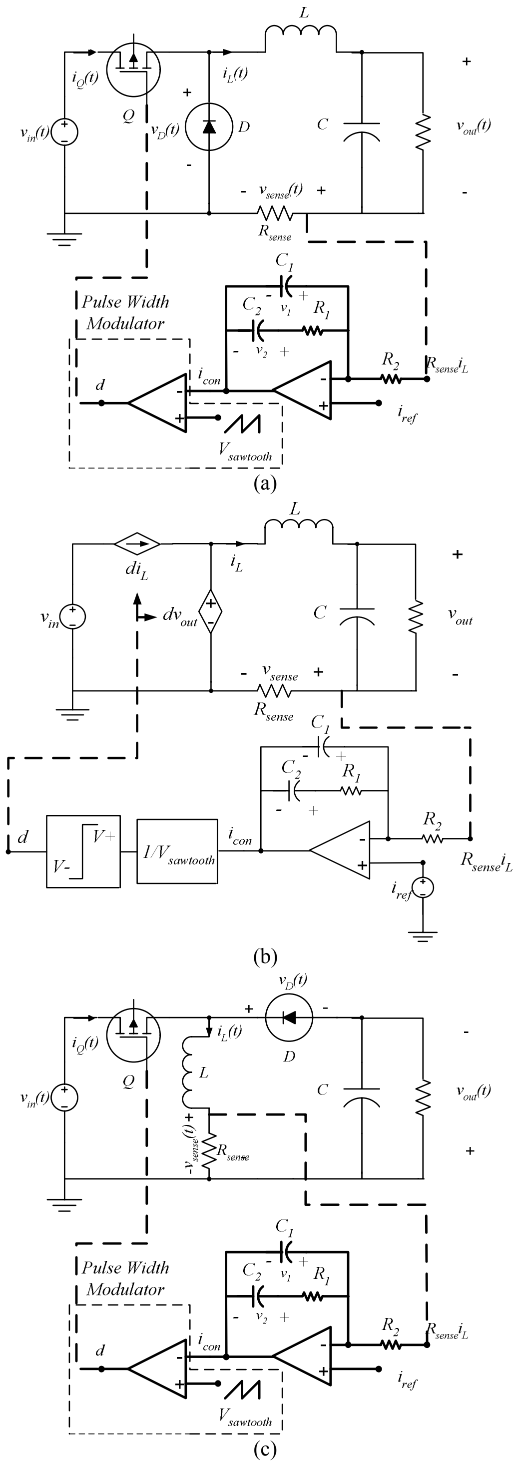
where

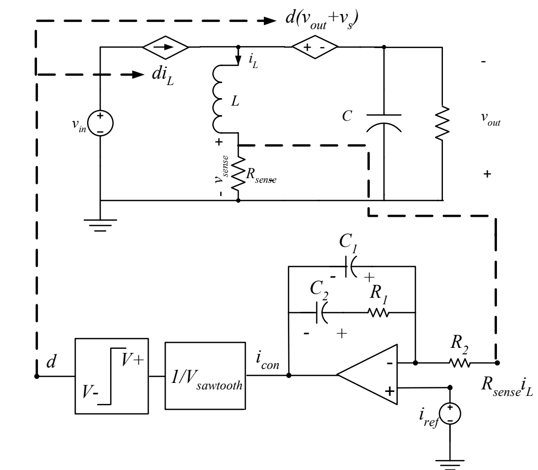
$$T_1 = A_2(sCR+1) + RA_5 - RA_1A_5$$

$$T_2 = (sL + A_3 + R_{sense})(sCR+1) - RA_1 + RA_1A_4 + R - RA_4$$

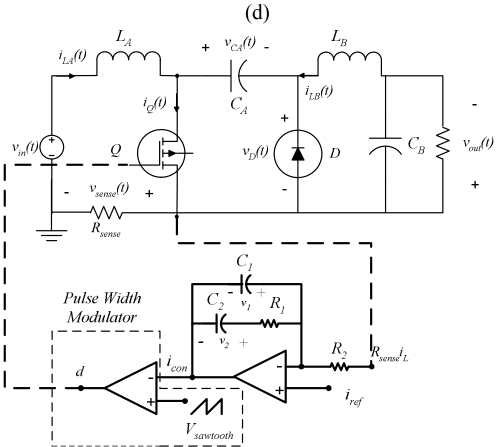
V. The Model For Various DC-to-DC Converters With Average Current Control

The proposed modeling technique can be applied to single switch converters other than the Boost converter. The model in Fig. 4 can be modified for converters such as Buck, Buck-Boost and Ćuk by replacing the averaged model of the Boost converter with the averaged model of the corresponding converter. Average current mode control is commonly used to control the continuous inductor current. Consequently the location of the sensing resister has to be changed depending on the DC-to-DC converter. The models for the Buck, Buck-Boost and Ćuk converters are shown in Fig. 6.





(d)



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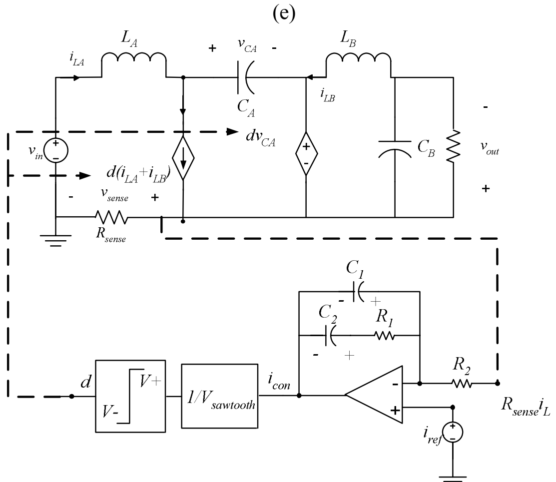


(f)

Fig. 6. (a) The Buck converter with average current control (b) The proposed model for the Buck converter with average current control (c) The Buck-Boost converter with average current control (d) The proposed model for the Buck-Boost converter with average current control (e) The Ćuk converter with average current control (f) The proposed model for the Ćuk converter with average current control.

VI. Model Verification

To validate the proposed model, a prototype Boost converter using the TL494 PWM control chip in average current control mode was constructed. The circuit parameters for both the model simulation and the prototype are listed in tables 1 and 2. To verify the proposed model, simulation results from the proposed model were compared to the experimental results (Fig. 7 - Fig.11).

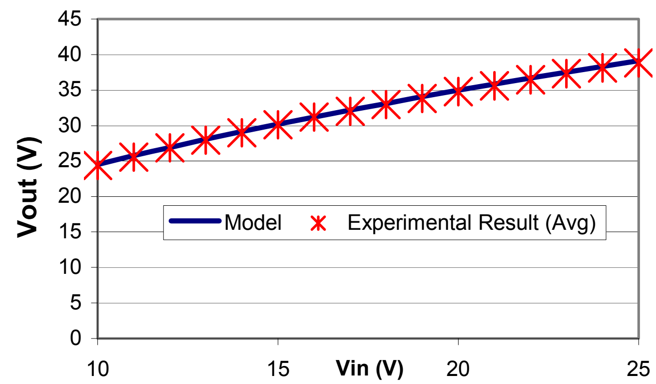
TABLE I. Parameters of the Boost Converter

Input Voltage [V_{in}]	Output Voltage	Inductor	Output Capacitor	Switching Frequency	Load Resistor [R_{load}]	Sensing Resistor [R_{sense}]
15V	30V	0.6mH	40µF	100kHz	62Ω	0.27Ω

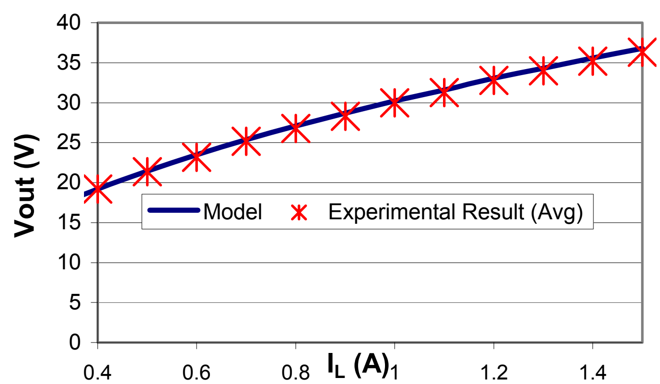
TABLE II. Parameters of the Averaged Current Control Scheme

i_{ref}	C_1	C_2	R_1	R_2	$V_{sawtooth}$
1A	82pF	150nF	10kΩ	2.5kΩ	3V

A. Steady-state characteristics



(a)



(b)

Fig. 7. Steady-state characteristics: (a) Vout vs Vin [Iin=1A]. (b) Vout vs Iin [Vin=15V].

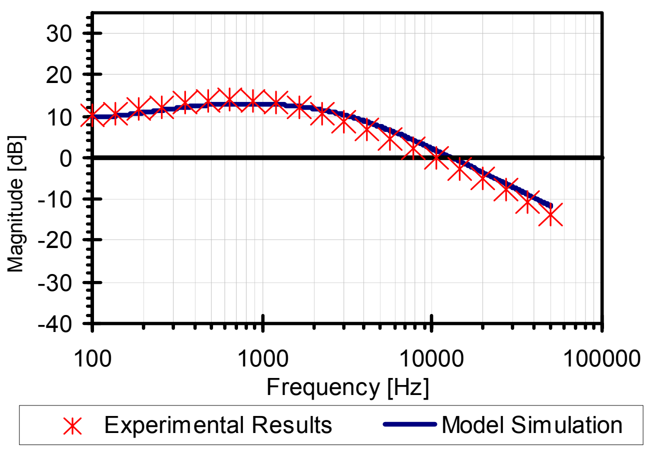
Fig. 7(a) shows the steady-state relationship between V_{in} and V_{out} (V_{in} varies from 10V to 25V) while the input current is kept constant at 1A and Fig. 7(b) shows the relationship between V_{out} and I_L (I_L varies from 0.4A to 1.5A) while the input voltage is kept constant at 15V. The results obtained by performing a DC sweep on the model agree very well with the experimental measurements. Equations (13) and (14) express the steady-state output voltage and input current in terms of the duty cycle. When the inductor current is kept constant, the steady-state relationship between the input voltage and the output voltage is nonlinear. Also when the input voltage is kept constant, the relationship between the input current and the output voltage is nonlinear as well. From Fig. 7, the nonlinearity can be observed. These nonlinearities are due to the *1/(1-D)* term in the steady-state equations for the output voltage and input current.

$$V_{out} = \frac{V_{in}}{1-D} \tag{13}$$

$$I_L = \frac{V_{in}}{(1-D)^2 R} \tag{14}$$

B. *Small signal model verification*

The $\hat{i}_{ref}$ -to- $\hat{i}_L$ and $\hat{i}_{ref}$ -to- $\hat{v}_{out}$ responses were obtained using the method of injecting a small sinusoidal disturbance into the reference signal i_{ref}. To verify this model, the frequency response of the simulations using the proposed model was compared to the measurements obtained from the prototype using a network analyzer. The Bode plot comparisons are shown in Fig. 8 and Fig. 9. The dashed line represents the experimental measurements and solid line is the simulation results obtained from the model. The simulation results from the model are very similar to the experimental measurements for up 50 kHz which is half the switching frequency (Fig. 8 and Fig. 9). Though, the agreement between the model and the experiment is quite good, especially for low frequencies, the difference tend to get larger as the frequency increases. Some of the discrepancy is due to the nature of practical components such as the inductor and capacitor, whose values are dependent on factors such as temperature, current and frequency.



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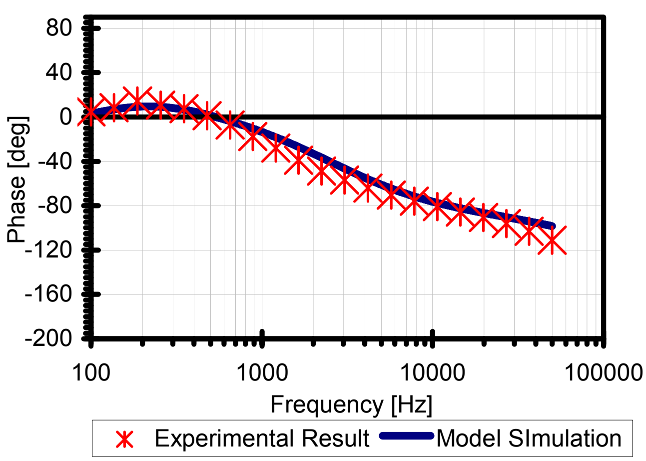
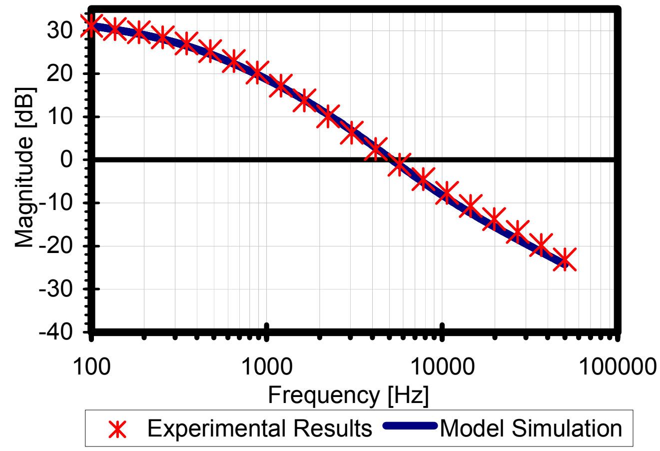
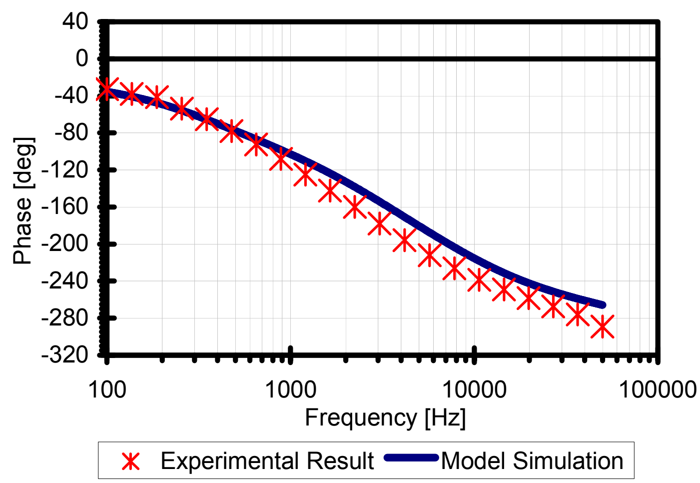


(b)

Fig. 8. Bode plots of the transfer function $\hat{i}_L / \hat{i}_{ref}$



(a)



(b)

Fig. 9. Bode plots of the transfer function $\hat{v}_{out} / \hat{i}_{ref}$

C. *Largel signal model verification*

The simulation and measured large signal waveforms for output voltage and input current were compared between the model and the prototype. A phase plane plot is useful in analyzing the large signal characteristics of the model [1]. The parameters of the model are that listed in tables 1 and 2 except that the reference current signal is stepped between 0.5A and 1A. From Fig 10, it can be seen that the input current responds quickly when the control signal i_{ref} is stepped from 0.5A to 1A and the output voltage changes quite slowly. Similar response of the input current and output voltage occurs when the control signal i_{ref} is stepped down from 1A to 0.5A. The faster response of the input current due to the change in reference signal i_{ref} is reasonable since the input current i_L is directly controlled by the average current control scheme.

A comparison of the dynamic responses of the simulation using the proposed model and prototype measurements was conducted. Fig. 11 illustrates the transient response when a step change of 0.5A to 1A in the current reference is applied. It is shown in Fig. 11 that the transient waveform obtained from the simulation and experimental results matches quite well with each other. The experimental results presented in Fig. 11 and Fig. 12 are averaged to remove the switching ripple. When i_{ref} is stepped from 0.5A to 1A, the output voltage increases from 22V to 30V and the input current changes from 0.5A to 1A. This shows that the proposed model can successfully predict the response of the Boost converter with average current control, even with a large change in the control signal i_{ref}.

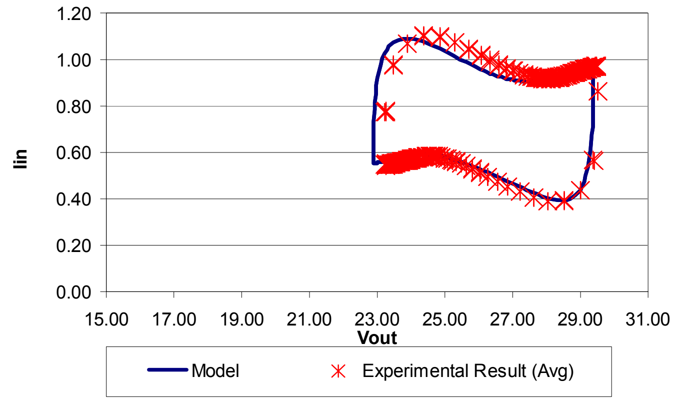
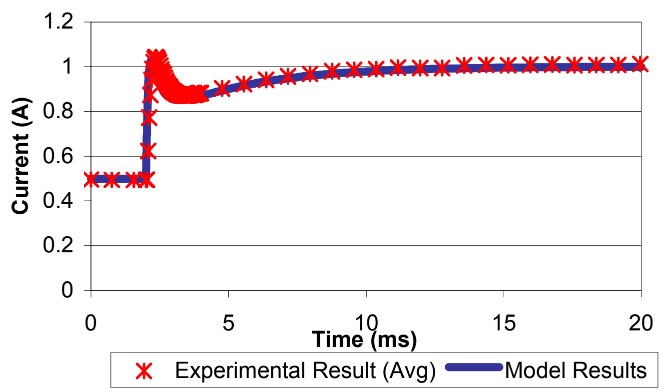
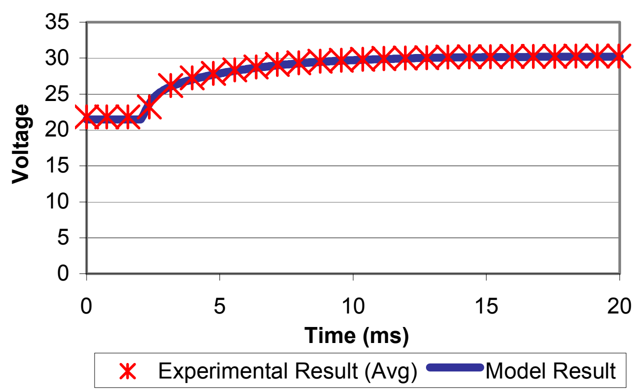


Fig. 10. Phase plane plot when the control signal i_{ref} is stepped between 0.5A and 1A – solid line: model simulation, dashed line: experimental results



(a)



(b)

Fig. 11. Simulation and experimental results when a step change in the reference current signal is applied: (a) Transient input current response when the reference current is stepped from 0.5A to 1A. (b) Transient output voltage response when the reference current is stepped from 0.5A to 1A.

VII. Conclusion

A unified small and large signal model for single switch converters using average current control has been presented in this paper. This model is obtained by combining the averaged model of the single switch converter for the power stage and the average current control scheme with the PWM circuit model. The average current control scheme and the PWM model determines the duty cycle value that generates the converter model waveforms such as input current and output voltage. The model method has been verified by using a prototype Boost converter.

The small and large characteristics of the proposed model were compared to the experimental results. In this paper, the small signal behavior was analyzed by examining the frequency response of the current reference to the output voltage and the input current. The large signal behavior was analyzed by both phase plane plot and large signal transient response when a step change is applied to the current reference. The simulated waveforms were compared to the experimental results obtained from the prototype. The measured small, large and steady-state signal responses of the prototype circuit verified the accuracy of the model.

The model presented in this paper can accurately predict the system's dynamic response which will aid designers in the control design. Its compatibility with simulation software allows easy analysis of steady-state, small and large signal characteristics. With this model, accurate predictions of the behavior of the real circuit are obtained and an excellent understanding of the circuit is achieved.

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