A New Sliding Mode Like Control Method for Buck Converter

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Abstract - **Sliding mode control is a robust control method with guaranteed large signal stability and good dynamic response. Despite its advantages, sliding mode control for DC-DC converters suffers from non-zero steady state error and variable switching frequency that depends on the converter parameters. This paper proposes a simple sliding mode like control (SMLC) technique for the buck converter that has a fixed switching frequency and zero steady state error. The response of this controller is robust and can be defined directly in the time domain. In addition, fuzzy logic implementation of the proposed SMLC is provided. Experimental results show that the proposed SMLC has good first-order dynamic performance and no steady state error.**

I. INTRODUCTION

In recent years, sliding mode control (SMC) has been investigated as a new control method for DC-to-DC converters. It has the advantages of high robustness, guaranteed first-order response and large signal stability [1]. The design of sliding mode controller does not require an accurate model of the system.

Despite these advantages, there are a number of problems with sliding mode control. The switching frequency of power converters under sliding mode control is not fixed and depends on a number of variables including the input supply voltage and the equivalent series resistance of the output filter capacitor. It is easier to design a converter with fixed switching frequency due to the timing considerations in transistor gate drive circuitry. In addition, sliding mode control also exhibits steady state error for the output voltage.

This paper presents a simple non-linear control scheme called sliding mode like control (SMLC) that has good features of sliding mode control and eliminates its problems. Specifically, the proposed controller has a robust first order response that is defined directly in the time domain as with sliding mode control. However, this controller also has fixed switching frequency and provides zero steady state error. This control scheme is developed for the buck converter topology and digital implementation of the controller is considered. It has been noted that fuzzy logic can give a sliding mode like response [2]. This paper also provides an explanation of how this is possible by implementing the proposed sliding mode like control in fuzzy logic.

The organization of this paper is as follows. Section II gives a review of sliding mode control for buck converter. Section III describes the proposed sliding mode like control method. Section IV discusses its implementation in fuzzy logic. Section V gives the experimental results of the sliding mode like control method and its fuzzy logic implementation. Section VI gives conclusions.

II. REVIEW OF SLIDING MODE CONTROL

When sliding mode control is applied to the buck converter shown in Fig. 1, the desired first-order response with time constant $\frac{1}{K}$ can be expressed in the time domain by a first-order differential equation

$$K \cdot e + \dot{e} = 0 \tag{1}$$

where $e(t) = v_o(t) - V_{ref}$, $v_o(t)$ is the output voltage, V_{ref} is reference voltage. $\dot{e}$ is the derivative of the error. The response of equation (1) describes a sliding line σ=0 as shown in Fig. 2, where σ is defined as:

$$\sigma = K \cdot e + \dot{e} \tag{2}$$

It is noted that the states of the system can be expressed in terms of e and $\dot{e}$ rather than the inductor current and the capacitor voltage [1]. Therefore, σ=0 describes the desired trajectory of the system in the state space.

For the buck converter shown in Fig. 1, sliding mode control is achieved by turning the active switch S on when σ<0 (below the sliding line) and off when σ>0 (above the sliding line), as shown in Fig. 2. Usually, hysteresis is needed to limit the switching frequency in a practical converter, which would otherwise be infinitely large. The action of the hysteresis block can be described as following: when σ becomes less than the lower hysteresis limit, l_l, the switch S is turned on; when σ becomes greater than the upper hysteresis limit, l_u, the switch S is turned off. A block diagram of a sliding mode controller is given in Fig. 3. Computing σ requires a method of approximating the derivative of the error, which is done by using a non-ideal differentiator. The non-ideal differentiator is implemented by using a first-order high pass filter with a transfer function given by $ce(s) = 2\pi f_0 \frac{s}{s + 2\pi f_0} e(s)$, where f_0 is the corner frequency of the high pass filter.

Sliding mode control has the advantage of first-order response for large reference changes and start up. This response can be directly defined in the time domain. There is no overshoot in the output voltage for reference changes, line changes, or load changes. The response is guaranteed for large signal changes, which are not covered by the small

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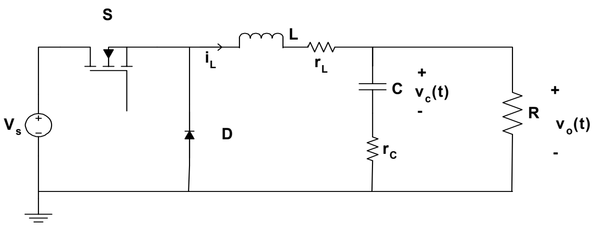


Fig. 1 Block diagram of buck converter

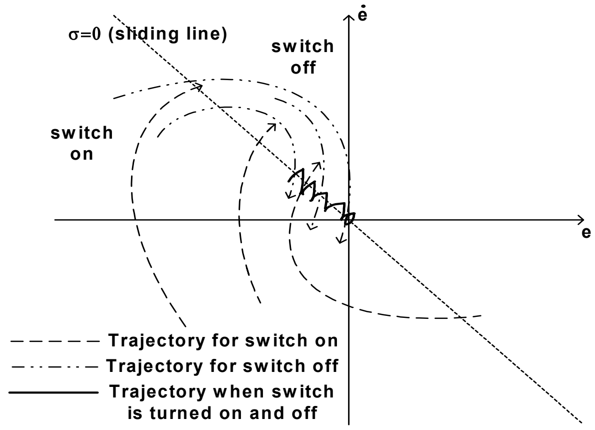


Fig. 2 Examples of phase plane trajectories for buck converter

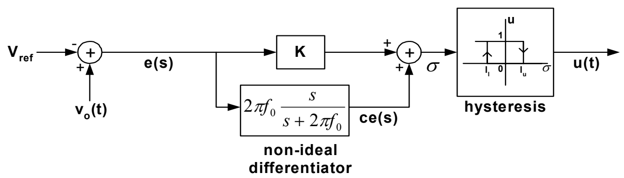


Fig. 3 Block diagram of sliding mode controller

signal model. However, sliding mode control also has some undesirable traits: non-zero steady state error, and switching frequency depends on the capacitor ESR and the supply voltage.

III. THE PROPOSED SLIDING MODE LIKE CONTROL

In this paper, a new non-linear control scheme called sliding mode like control scheme is proposed (shown in Fig. 4). A voltage mode (direct duty ratio) digital control scheme is used, where the output voltage is sensed and sampled with period Ts. The duty cycle $u(t)$ of the fixed switching frequency converter is altered by SMLC. The inputs to the controller are the error $e(k)$ and the change of error $\Delta e(k)$, which is calculated by

$$\Delta e(k) = e(k) - e(k-1) \quad (3)$$

The output of the controller is the incremental change of the duty cycle $\Delta u(k)$. The incremental change of the duty cycle is then added to the previous value of the duty cycle $u(k-1)$,

$$u(k) = \Delta u(k) + u(k-1) \quad (4)$$

Equation (4) is a digital approximation for integration. Saturation in the integration path prevents integrator wind-up and limits the duty cycle between 0 and 1 (shown in Fig. 4). In Fig. 4, z is the Z-transform variable and z^{-1} represents the unit time delay.

In sliding mode control of buck converter, a sliding line is given to define the desired response. The switch is turned on below the sliding line and off above it. This forces the response of the converter to follow the sliding line. In sliding mode like control, the duty cycle is increased when the system operates below the sliding line and decreased when the system operates above the sliding line. Along the sliding line, the change of duty cycle is zero. The magnitude of the change in duty cycle is greater further from the sliding line, up to some limit. This limit is reached at the upper and lower boundary lines. Therefore, the response of the system is forced to follow the sliding line to give the desired dynamic response as with sliding mode control. This idea is illustrated in Fig. 5.

To clarify this concept, let us consider the points labelled A through H in Fig. 5. The changes of duty cycle at point A and point H are zero. Point A is a stable point because the rate of change of error is zero, while point H is not stable because the change of error is non-zero. Therefore, the system will move from point H eventually. For example, it is assumed that the maximum change of duty cycle is ±20%. At points C and D, the changes of duty cycle are –20%. At points F and G, the changes of duty cycle are +20%. At point B, the change of duty cycle is –10%, and at point E, it is +10%, since these points lie half way between the sliding line and boundary lines.

The sliding mode like control described above will now be precisely defined. The rate of change of error $\dot{e}$ can be approximated as

$$\dot{e} \approx \frac{e(k) - e(k-1)}{T_s} = \frac{\Delta e(k)}{T_s} \quad (5)$$

The inputs to the sliding mode like control block in Fig. 4 are e' and $\Delta e'$ (note that sample numbers will no longer be included in the notation; dependence on k is implicit). They are related to e and Δe by the gains G_1 and G_2 that serve to scale the inputs

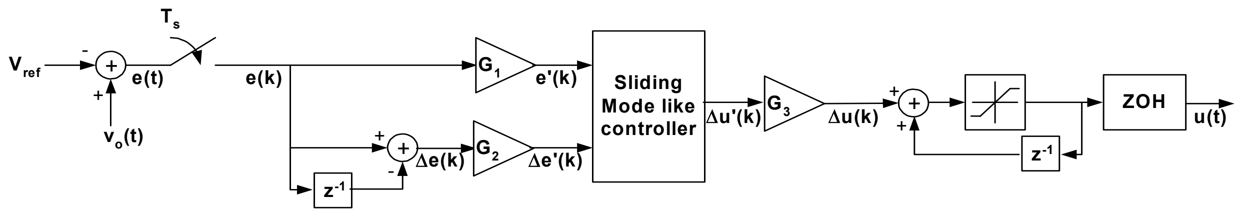


Fig. 4 Block diagram of the proposed sliding mode like controller

$$e' = G_1 \cdot e \quad (6)$$

$$\Delta e' = G_2 \cdot \Delta e \quad (7)$$

This scaling is useful in the case where digital hardware used to implement the control algorithm has limits on the numerical range it can handle.

The output of the sliding mode like control block is the normalized change of duty cycle $\Delta u'$. It is scaled by G_3 to give the actual change of duty cycle

$$\Delta u = G_3 \cdot \Delta u' \quad (8)$$

The dynamic response is defined as a sliding line, which is the same as sliding mode control (shown in equation (1)).

In terms of the normalized inputs, the sliding line can be expressed as:

$$\Delta e' + K' \cdot e' = 0 \quad (9)$$

The relationship between K' and K is found as

$$K' = \frac{K \cdot T_s \cdot G_2}{G_1} \quad (10)$$

where T_s is the sampling time.

Refer to Fig. 6, a unit vector $\vec{m}$ is defined to be parallel to the sliding line. The components of this vector $\vec{m} = (m_1, m_2)$ are given by

$$\vec{m} = (m_1, m_2) = \left(\frac{-1}{\sqrt{1+K'^2}}, \frac{K'}{\sqrt{1+K'^2}} \right) \quad (11)$$

A normal $\vec{n}$ to the sliding line is now determined by rotating $\vec{m}$ by -90°. By doing so, we can find: $\vec{n} = (m_2, -m_1)$. Let h_0 be the perpendicular distance between the sliding line and each of the upper and lower boundary lines. The perpendicular distance h from the sliding line to the point representing the controller input, can be calculated by using the projection of a vector representing the input point onto $\vec{n}$:

$$h = m_2 \cdot e' - m_1 \cdot \Delta e' \quad (12)$$

The controller output (the normalized change of duty cycle) is determined by

$$\begin{aligned} &if \quad h > h_0 \quad then \quad \Delta u' = -1 \\ &if \quad h < -h_0 \quad then \quad \Delta u' = 1 \\ &if \quad -h_0 < h < h_0 \quad then \quad \Delta u' = -\frac{h}{h_0} \end{aligned} \quad (13)$$

It should be noted that for small signals that do not cause saturation in the output integration path (refer to Fig. 4) and that are applied when the system is in the steady state (i.e. when the controller inputs are very close to the zero error and zero change of error values), the controller is essentially a digital PI controller. In this case, the small signal model of buck converter [3,4] can be used to assess the small signal stability of the control loop. The following analysis will reveal the relationship between the controller parameters, switching frequency, and stability.

It is assumed that the applied signals are small enough such that $-h_0<h<h_0$ and the saturation block in the output integrator does not reach its saturation limits. In this case,

$$\Delta u = -\frac{m_2 \cdot G_1 \cdot G_3}{h_0} e + \frac{m_1 \cdot G_2 \cdot G_3}{h_0} \Delta e \quad (14)$$

It is noted that the transfer function for a digital PI controller is in the form

$$\frac{U(z)}{E(z)} = \frac{m \cdot z + n}{z - 1} \quad (15)$$

where m and n are parameters. This transfer function can be written in the time domain as

$$u(k) = (m+n)e(k) - n(e(k) - e(k-1)) + u(k-1) \quad (16)$$

Therefore,

$$\Delta u(k) = (m+n) \cdot e(k) - n \cdot \Delta e(k) \quad (17)$$

By comparing equations (14) and (17), we can see that

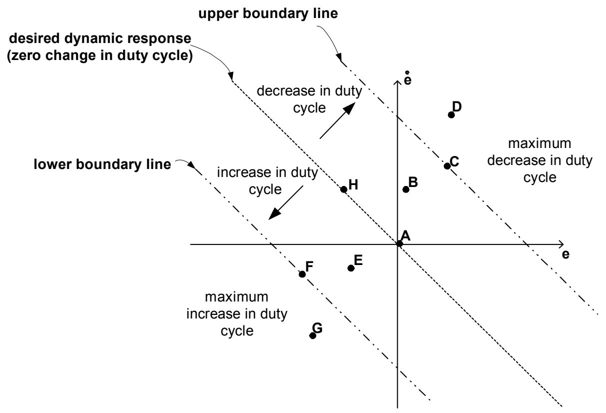


Fig. 5 Basic idea for sliding mode like control

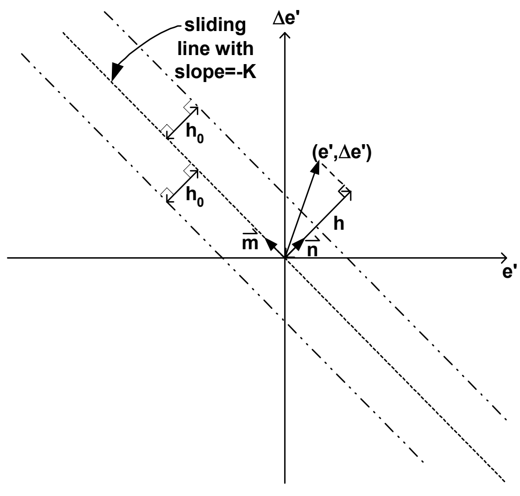


Fig. 6 Illustration of control scheme

$$m+n=-\frac{m_2\cdot G_1\cdot G_3}{h_0} \quad (18)$$

$$-n=\frac{m_1\cdot G_2\cdot G_3}{h_0} \quad (19)$$

Therefore, the controller gain is $\frac{G_3}{h_0}(-m_2\cdot G_1+m_1\cdot G_2)$ and the location of the controller zero z_0 in the z-domain is

$$z_0=\frac{m_1\cdot G_2}{-m_2\cdot G_1+m_1\cdot G_2} \quad (20)$$

By substituting (10), (11) into (20), the controller zero can be expressed as

$$z_0=\frac{1}{1+K\cdot T_s} \quad (21)$$

This means that the sampling frequency, desired dynamic response and controller zero are coupled. There are constraints on appropriate values of the sampling/switching frequency and desired dynamic response to maintain small signal stability.

IV. FUZZY LOGIC IMPLEMENTATION OF SLIDING MODE LIKE CONTROL

The control scheme of section III can be implemented in fuzzy logic to give a fuzzy logic controller (FLC) that has sliding mode like characteristics. To do this, the sliding mode like control block in Fig. 4 is replaced by a two-input, single-output FLC. It is noted that a two-input, single-output relation with a one-to-one mapping of input pairs to the output can be visualized as a surface in a three dimensional space. By placing certain constraints on the membership functions and inference mechanism of the FLC, two important properties can be realized that allow the sliding mode like controller described in section III to be transferred to the FLC [5]. These two important properties of the fuzzy controller are:

Property 1: Each rule gives the controller output when the inputs to the controller have full membership to the fuzzy sets in the antecedent of that rule. The controller output in this case is equal to the consequent of the rule. If the input-output relationship of the controller is visualized as a control surface, this means that each rule will define a specific point on the control surface.

Property 2: If the consequent of each of the four active rules lie in a plane on the control surface, then all points calculated by the fuzzy controller using these rules will lie in that plane.

The necessary constraints on the fuzzy controller to realize these properties are as follows. The input membership functions are triangular except for the left most and right most membership function (illustrated in Fig. 7). The variable x in Fig. 7 is a generic variable; x could represent the error e or the change of error Δe. The membership functions are arranged so that at most two have non-zero membership (are active) for any value of the input and are not necessarily evenly distributed as they are shown in Fig. 7. Furthermore, the sum of the membership for all active fuzzy sets on each input is exactly one.

With this description of the membership functions, if $x_1 < x < x_n$, membership can be calculated as:

$$\mu_{Ak+1}(x)=\frac{x-x_k}{x_{k+1}-x_k} \quad (22)$$

$$\mu_{Ak}(x)=1-\frac{x-x_k}{x_{k+1}-x_k}=1-\mu_{Ak+1}(x) \quad (23)$$

where μ_{Ak} is the membership of x to A_k, μ_{Ak+1} is the membership of x to A_{k+1}, x_k is the point where x has full membership to A_k, and x_{k+1} is the point where x has full membership to A_{k+1}. There are two exceptions where equations (22) and (23) cannot be used to calculate membership. If $x < x_1$, then $\mu_{A1}(x)$ = 1 and membership is zero for all other fuzzy sets. If $x > x_n$, then $\mu_{An}(x)$ = 1 and membership is zero for all other fuzzy sets.

The proposed fuzzy controller is defined to be a Sugeno-type fuzzy logic controller. This means the output membership functions are singletons (crisp values).

This fuzzy controller has rules of the form "If e' is A_k and $\Delta e'$ is B_k, then $\Delta u' = \Delta u'_{AkBk}$", where A_k and B_k are fuzzy sets on the error and change of error inputs respectively. The "and" operation in the rule antecedent is performed by multiplication. Active rules are combined by the "or" operation, which is accomplished by addition. For example, $(e', \Delta e')$ is the input to the controller, where e' belongs to A_k and A_{k+1}, $\Delta e'$ belongs to B_k and B_{k+1}. Because for each input at most two fuzzy sets have non-zero membership, at most four rules are activated. These rules are:

(1) If e' is A_k and $\Delta e'$ is B_k, then $\Delta u' = \Delta u'_{AkBk}$.

(2) If e' is A_{k+1} and $\Delta e'$ is B_k, then $\Delta u' = \Delta u'_{Ak+1Bk}$;

(3) If e' is A_k and $\Delta e'$ is B_{k+1} then $\Delta u' = \Delta u'_{AkBk+1}$;

(4) If e' is A_{k+1} and $\Delta e'$ is B_{k+1} then $\Delta u' = \Delta u'_{Ak+1Bk+1}$.

The output of this FLC is the weighted sum of all the activated rules and is given by

$$\Delta u'=(\mu_{Ak}\cdot\mu_{Bk})\cdot\Delta u'_{AkBk}+(\mu_{Ak+1}\cdot\mu_{Bk})\cdot\Delta u'_{Ak+1Bk} +(\mu_{Ak}\cdot\mu_{Bk+1})\cdot\Delta u'_{AkBk+1}+(\mu_{Ak+1}\cdot\mu_{Bk+1})\cdot\Delta u'_{Ak+1Bk+1} \quad (24)$$

The sliding mode like controller described in section III can be transferred to this FLC by applying property 1. It should be noted that the input-output relationship of the FLC

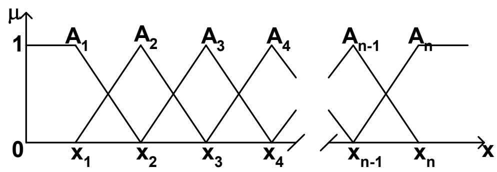


Fig. 7 Input membership function of the proposed fuzzy logic controller

may not be exactly the same as the original. It will only be the same at the points defined by the rules. However, if enough rules (and hence input membership functions) are used and are properly distributed, the relationships can be similar enough. Therefore, the proposed FLC can provide nearly the same control of SMLC. This means that the number and distribution of membership function must be chosen carefully. Rules are initialized by setting the rule consequent equal to the result of the control relation given in equations (12) and (13), where the inputs e' and $\Delta e'$ have full membership to the fuzzy sets in the rule antecedent (full membership only happens at one point for triangular membership functions). For the membership functions at the extremes, the point where unity membership occurs first is used. For example, it is supposed that the membership to A_k is equal to 1 at e_k', and the membership of B_k is equal to 1 at $\Delta e_k'$. It is noted that the subscript k is used to be a specific number. Thus, e_k', $\Delta e_k'$, and $\Delta u_k'$ are actual numbers. The value of h calculated in (12) and (13) is used to find the value of $\Delta u_k'$. For the rule "If e' is A_k and $\Delta e'$ is B_k then $\Delta u' = \Delta u'_{AkBk}$", the value of $\Delta u'$ is set to be $\Delta u_k'$.

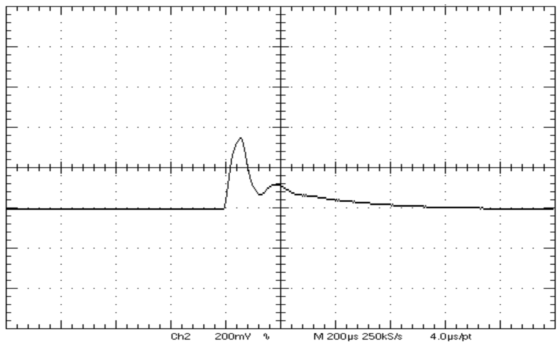
V. EXPERIMENTAL RESULTS

A Buck converter with the proposed sliding mode like controller and its fuzzy implementation was built to verify the proposed methods. An FPGA (200K gates) was used to implement the sliding mode like controller and fuzzy logic controller. The PWM signal is calculated and generated by the control algorithms inside FPGA. The parameters of Buck converter are listed as following: input voltage $V_{in} = 5V$, output voltage $V_o = 2.5V$, rated power = 25 W, $L = 1uH$, $C = 220uF$, ESR=1mΩ, R_L=2mΩ, where ESR is the equivalent series resistor of the output capacitor and R_L is the winding resistor of the inductor. The switching/sampling frequency is 400Khz.

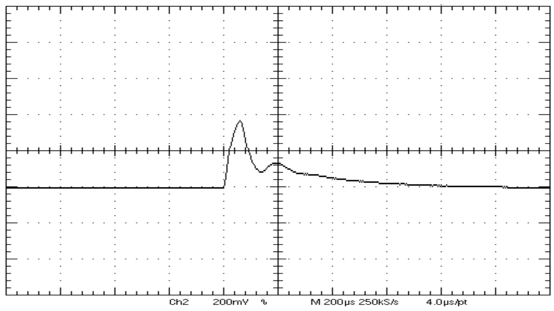
Fig. 8-10 shows the dynamic response of proposed sliding mode like controller and its fuzzy logic implementation under input voltage change, load variation and step reference change. From the experimental results, it is shown that the proposed SMLC and its fuzzy logic implementation have no steady state error. There is no overshoot in the output voltage during the transient time. Good first order dynamic response is achieved.

VI. CONCLUSIONS

This paper has proposed a digital sliding mode like controller and its fuzzy logic implementation for the buck converter. The proposed control scheme has the fixed switching frequency and provides zero steady state error. Furthermore, this controller also gives a robust first-order response that is defined directly in the time domain as with sliding mode controller. Experimental results show that the proposed SMLC and its fuzzy implementation have good steady state and dynamic response. All these benefits indicate that such schemes can be an attractive alternative to the classic controller in power converter applications where high dynamic performance is preferred.

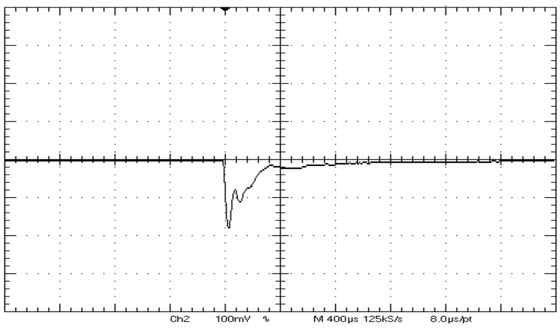


(a) The proposed sliding mode like controller

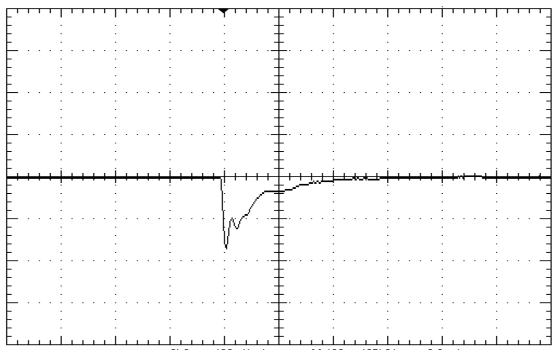


(b) fuzzy logic implementation of SMLC

Fig. 8 Output voltage response to input voltage change from 5V to 6V (X axis: 200us/div, Y axis: 200mv/div)

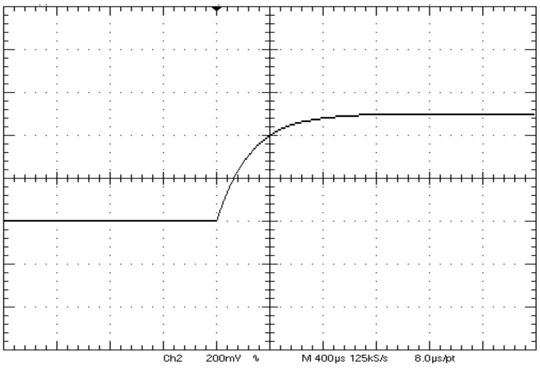


(a) The proposed sliding mode like controller

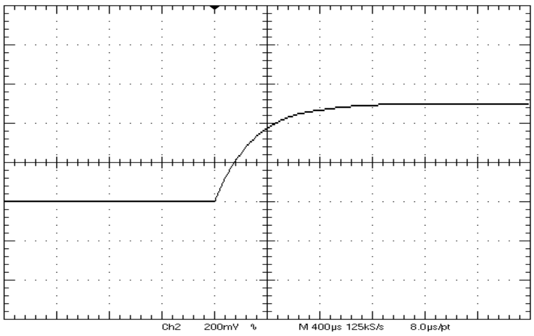


(b) fuzzy logic implementation of SMLC

Fig. 9 Output voltage response to load current change from 5A to 10A (X axis: 400us/div, Y axis: 100mv/div)



(a) The proposed sliding mode like controller



(b) fuzzy logic implementation of SMLC

Fig. 10 Output voltage response to reference voltage change from 2.5V to 3V (X axis: 400us/div, Y axis: 200mv/div)

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