A Large Signal Dynamic Model for Single-Phase AC-to-DC Converters with Power Factor Correction

Yun W. Lu, Wangfeng Zhang, Yanfei Liu
Department of Electrical and Computer Engineering
Queen's University at Kingston, Canada
Email: yun.lu@ece.queensu.ca, wanfeng.zhang@ece.queensu.ca, yanfei.liu@ece.queensu.ca

Abstract— **This paper presents a model for average current control that can be applied to DC-to-DC converters and AC-to-DC power factor correction (PFC) circuits. The proposed DC-to-DC model consists of two parts: 1) an averaged DC-to-DC converter topology with all the switching elements replaced by dependent sources 2) an average current control scheme with a Pulse Width Modulation (PWM) model, which determines the duty cycles. Similarly, the AC-to-DC PFC model is obtained by combining an averaged Boost converter model with the PFC control scheme using average current control.**

To verify the proposed model, simulated results were compared to experimental waveforms. The experimental results demonstrate that the model can correctly predict the steady-state and large signal dynamic behavior for average current controlled DC-to-DC and AC-to-DC PFC converters.

I. INTRODUCTION

Average current control is a control scheme that is commonly used for DC-to-DC converters and AC-to-DC power factor correction (PFC) circuits. It has several advantages over peak current control such as the elimination of the external compensation ramp, increased gain for DC and low frequencies and improved immunity to noise in the sensed current signal over peak current mode control and lower input current THD [2,3,10]. Simulation software packages such as Spice include integrated circuits modules that perform average current control. However, the problem with these types of circuit simulations is their long computation time, which is caused by several reasons:

1. Due to the complexity of the control algorithms and switching elements, long computational times will be required to generate the simulation waveforms. Particularly when the converter topologies and control circuitry are not linear [7].
2. Long computation time occurs when the time constant of the converter is much greater than the switching period. In order to show how these types of converters behave at steady-state, the operating time of the simulated circuit will be quite long.
3. Numerous switches in a topology will also increase the computation time. In such cases, there are many diodes and MOSFETs switching at high frequencies and multiple control circuitries. As a result, the overall computation time for obtaining simulation waveforms at steady-state can drastically increase due to the increased number of switching elements and overall complexity.

In order to reduce the long computation times, the complex converter circuit can be replaced with a simple equivalent model.

In the past, several technical papers have addressed average current control modeling for DC-to-DC converters [3-5]. The models presented in [4,5] only deals with the analysis of the small signal characteristics. Though [3] does address some large signal issues, it cannot be applied to converters with nonlinear transfer functions such as the Boost, Buck-Boost and Ćuk converters. With the method presented in [3], for nonlinear converters, only the small signal model can be obtained by using state-space averaging which involves perturbing and linearizing around an operation point.

In addition several papers have addressed modeling AC-to-DC PFC converters [10]-[12]. Similarly, the majority only deals with small-signal analysis. Since the operation of a PFC circuit is large-signal by nature, a large signal model is needed. Furthermore, a large signal model is important for situations where switching circuit simulations take so long it becomes impractical. Although [10] and [12] proposed large-signal models for PFC circuits, unfortunately they both have some drawbacks. Paper [12] cannot be applied to PFC converters operating in continuous conduction mode (CCM) and [10] involves complicated transfer functions and it does not include a voltage loop.

In this paper, a new method of modeling large-signal dynamic characteristics of DC-to-DC and AC-to-DC PFC converters is proposed. This method takes advantage of advanced simulation software by utilizing the circuit representation of the model rather than an equation form.

The proposed model uses the averaged converter method to generate a non-switching model for the power stage [1]. It utilizes simulation tools such as SPICE to generate the large signal waveforms.

The proposed method has several advantages:

1. A simplified circuit model is used instead of complex transfer functions and/or differential equations.
2. This model can be conveniently used to determine the steady-state and large-signal dynamic performance of DC-to-DC and AC-to-DC PFC converters for all continuous mode operating conditions.
3. The proposed model is intuitive since the topology of the model is very similar to the topology of the circuit.
4. The model of the power stage is non-switching therefore the computation times are greatly reduced.

In this paper, the averaged model will be derived for the Boost topology in section II. In section III, the average

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current control transfer function for duty cycle is formulated. Section IV will deal with the DC-to-DC converter model, which consisted of the averaged model of the Boost converter combined with the average current control scheme. Section V discusses the AC-to-DC PFC model. In section VI, the validity of the model is verified through experimental analysis of an averaged current controlled Boost converter and an AC-to-DC Boost PFC converter with average current control. The experimental waveforms were compared to the model simulations for large signal dynamic response. Section VII is the conclusion.

II. THE AVERAGED MODEL

As mentioned in section I, replacing the circuit with a simpler converter model can reduce the computation time. Simplification of the model is gained by replacing the MOSFET and diode with non-switching dependent sources to produce an averaged non-switching converter model [1]. The MOSFET is modeled by a controlled current source, whose value is i_Q, the averaged value of the current through the MOSFET during each switching period. The diode is modeled by a controlled voltage source, whose value is v_G, the average voltage across the diode for each switching period. Fig. 1(b) illustrates the Boost converter with an added current sensing resistor. As long as the sensing resistor is small, its effect on Boost converter behavior is negligible. Fig. 1(a) shows the waveforms of the current through the MOSFET $i_Q(t)$ and the diode voltage $v_D(t)$ for one switching period. Low ripple assumption and ideal switches were assumed. The averaged Boost converter model can be derived from these waveforms [1]. The averaged model for a Boost converter with a sensing resistor is given in Fig. 1(c).

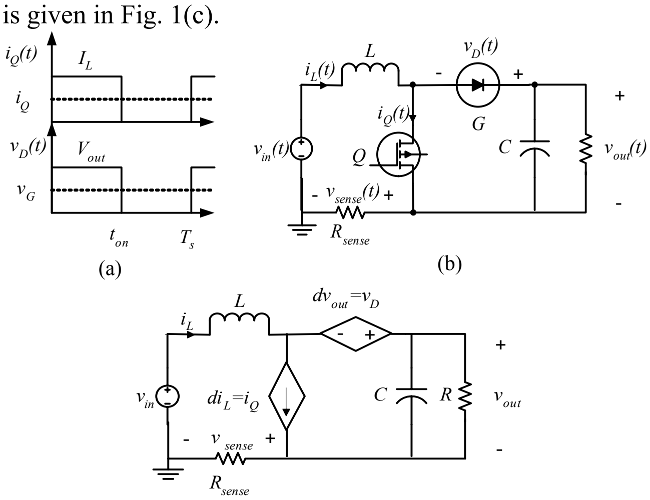


Fig. 1. (a) Switching and averaged waveforms of the Boost converter: the dashed line represents the averaged value of the waveform (b) Boost converter with R_{sense} (c) Averaged model for a Boost converter with R_{sense},

Fig. 1(b) shows that the averaged values of the current through the MOSFET and the voltage across the diode are related to the duty cycle. The duty cycle can be determined by (1), where t_{on} is the time the MOSFET is switched on and T_s is the switching period.

$$Duty = \frac{t_{on}}{T_s} = d \tag{1}$$

The controlled current source, which replaces the MOSFET in the averaged model, has a current value that is expressed in (2). The voltage value across the controlled voltage source that replaces the diode in the averaged model is expressed in (3). The current i_L is the current through the inductor when the MOSFET is on and v_{out} is the output voltage of the Boost converter. The values of the resistor, capacitor, inductor, and supply voltage remains unchanged in the model as compared to those of the Boost circuit because they are present for both on and off states of the MOSFET [1].

$$i_Q = \frac{t_{on}}{T_s} I_L = dI_L \tag{2}$$

$$v_D = \frac{t_{on}}{T_s} V_{out} = dV_{out} \tag{3}$$

For the averaged Boost converter model, it is shown that all the model waveforms can be determined by three variables: 1) the inductor current I_L when the MOSFET is on, 2) the output voltage of the converter V_{out} and 3) the duty cycle value d [1]. The averaged models for other types of converters such as Buck, Buck-Boost and the Ćuk converters can be derived using the same method [1].

III. MODEL OF AVERAGE CURRENT CONTROLLER

Average current control method is often used for applications where tight regulation of the current is needed, such as in PFC [8]. In average current control, the control variable is the reference current v_{iref} and it is compared to the sensed current signal. The sensed current signal is expressed in (4).

$$v_{sense} = R_{sense} \times i_L \tag{4}$$

With the sensed current and the reference current value, it is possible to determine the duty cycle value for average current control. This duty cycle value can then be fed into an averaged model of the DC-to-DC converter to obtain the desired waveforms. Fig. 2 illustrates the average current control scheme with a pulse width modulator.

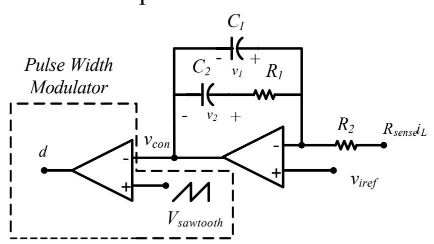


Fig. 2. Average current control scheme with a pulse width modulator

Since the feedback compensation for the op-amp is a low-pass network, the switching ripple in the inductor current i_L and v_{sense} can be neglected [10]. The sensed current signal v_{sense} is fed into R_2, which is connected to the inverting terminal of the op-amp. The non-inverting terminal of the op-amp is connected to the v_{iref}. The difference between v_{sense} and v_{iref} will be amplified by the compensation network to produce the signal v_{con}. v_{con} is then fed into the inverting input of the comparator to generate the duty cycle d. The non-inverting input of the comparator is a sawtooth signal with magnitude $V_{sawtooth}$ and frequency $f_{sawtooth}$. The output of the comparator d is the pulse width modulated (PWM) signal, which controls the duty cycle of the converter. The PWM circuit can be modeled as a constant multiplier $1/V_{sawtooth}$ and a voltage limiter which limits the duty cycle between d_{min} and d_{max}. Fig. 3 shows the PWM circuit model with average current control scheme.

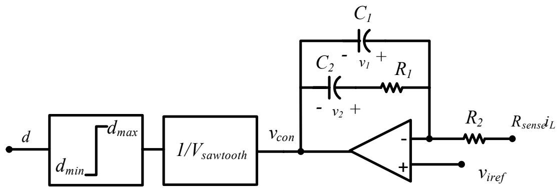


Fig. 3 Average current control scheme with the PWM model

From Fig. 3, the transfer function of duty cycle for average current control scheme can be derived as:

$$d(s) = \frac{1}{V_{sawtooth}}\left(I_{ref}(s) + H(s)(I_{ref}(s) - R_{sense}I_L(s))\right) \quad (5)$$

where $H(s)$ is the transfer function from the inverting terminal to v_d

$$H(s) = \frac{K_c(1 + s/\omega_2)}{s(1 + s/\omega_1)}$$

and K_c, ω_1 and ω_2 are defined by

$$K_c = \frac{1}{R_2(C_1 + C_2)}, \quad \omega_1 = \frac{C_1 + C_2}{R_1C_1C_2}, \quad \omega_2 = \frac{1}{R_1C_2}$$

This transfer function for duty is identical to what is derived in [3], [8]. The pole of $H(S)$ at the origin is used to increase DC gain of the current loop, the zero extends the crossover frequency and the high frequency pole is used to increase noise immunity [3], [4].

IV. THE BOOST CONVERTER WITH AVERAGE CURRENT CONTROL

Several method of modeling DC-to-DC converters with average current mode control is proposed in [3-5]. However, the problem with those proposed models for small and large signal analysis is their complexity. The derived small signal transfer functions are at least 4th order, depending on the DC-to-DC converter. In addition, for large signal analysis, the derived differential equations can be quite complex.

Fig. 4(b) shows the complete model for a DC-to-DC Boost converter with average current control. It consists of two parts: the averaged model of the Boost converter and the average current control compensator with the PWM circuit model. For the average current mode control scheme illustrated in Fig. 2 and describe by (5), the relationship between the input (v_{iref}, v_{sense}) and the output (v_{con}) is linear. As a result, using the proposed method, the current loop can easily be modeled since the average current control compensator remains unchanged in the circuit (Fig. 4a) as compared to that of the model (Fig. 4b).

When the model is in steady-state, the average magnitude of v_{sense} will be equal to the magnitude of the current reference v_{iref}. Therefore from (4), the relationship between the inductor current and the current reference can be expressed by

$$I_L = \frac{V_{iref}}{R_{sense}} \quad (6)$$

It can be observed from (6) that by using average current control, the magnitude and the shape of i_{ref} will determine the magnitude and shape of the input current i_L. The output voltage of the Boost converter can be expressed in terms of the reference current and the input voltage. By equating the input power to the output power of the Boost converter, the following equation can be derived.

$$V_{in}I_L = \frac{V_{out}^2}{R} + I_L^2R_{sense} \quad (7)$$

The relationship between the output voltage, input voltage and the current reference signal can be derived by substituting (6) into (7).

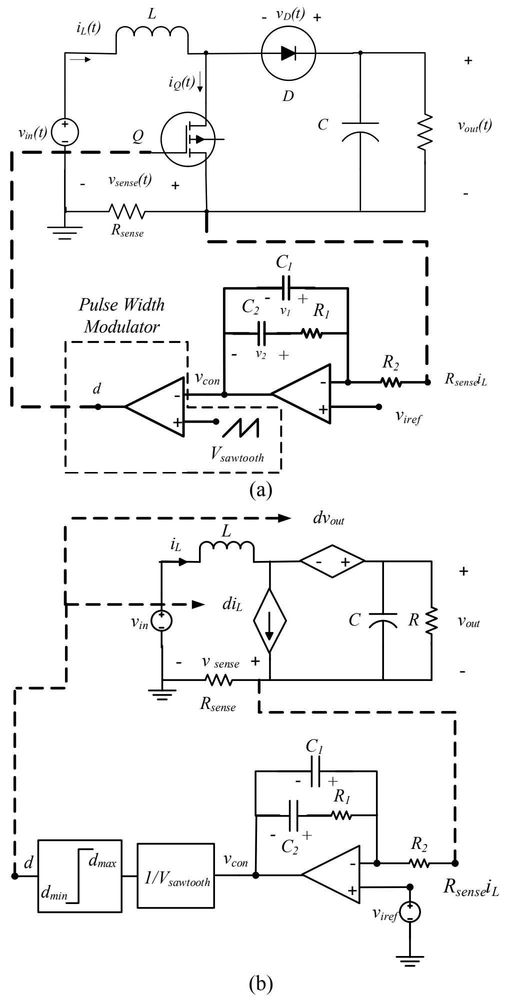


Fig. 3. (a) DC-to-DC Boost converter with average current control and PWM circuit (b) The proposed model: averaged DC-to-DC Boost converter model with average current control and PWM circuit model

$$\sqrt{\frac{RV_{iref}}{R_{sense}}\left[V_{in} - V_{iref}\right]} = V_{out} \quad (8)$$

It can be observed from (8) that the output voltage is dependent on the load resister, the sensing resister, input voltage and the current reference signal. Equation 8 shows that the steady-state relationship between the input voltage and the output voltage and the relationship between the reference current and the output voltage is nonlinear. The nonlinearity is due to the square root term that defines the output voltage.

V. AC-TO-DC PFC MODEL DESCRIPTION

Power factor correction (PFC) has become an important area of research in power electronics due to the increasing concerns of harmonic pollution in power systems caused by switching converters. An assortment of converters can be used for single-phase AC-to-DC PFC applications such as Buck, Boost and Flyback. One of the most commonly used is the Boost converter since its input current is continuous and can be actively shaped to the desired waveform [2]. Fig. 5 illustrates a single-phase Boost PFC converter with average current control.

The proposed model for the single-phase Boost PFC converter is illustrated in Fig. 6. It can be seen that it is very similar to the PFC circuit shown in Fig. 5. The major difference is that the averaged model of the Boost converter replaces the Boost converter for the power stage. In addition, the pulse width modulator (PWM) is modeled by a gain block and a limiter, which limits the maximum and minimum duty cycle.

In the AC-to-DC PFC model show in Fig. 6, the Boost converter and average current control model is identical to the DC-to-DC converter model illustrated in Fig. 4b. However, unlike the DC-to-DC converter model the Boost converter model show in Fig. 6 accepts an AC input voltage and includes a diode bridge.

The current loop utilizes average current control to control the magnitude and the shape of i_L. It forces i_L to follow the magnitude and shape of v_{iref}. Fig. 5 shows that the shape of v_{iref} is determined by the rectified input voltage signal V_{rec} and can be expressed by (9).

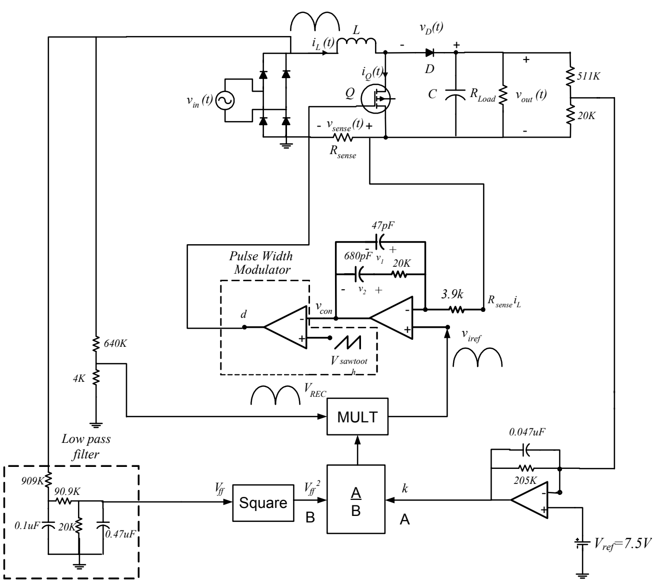


Fig. 5. Single-phase Boost PFC converter with average current control.

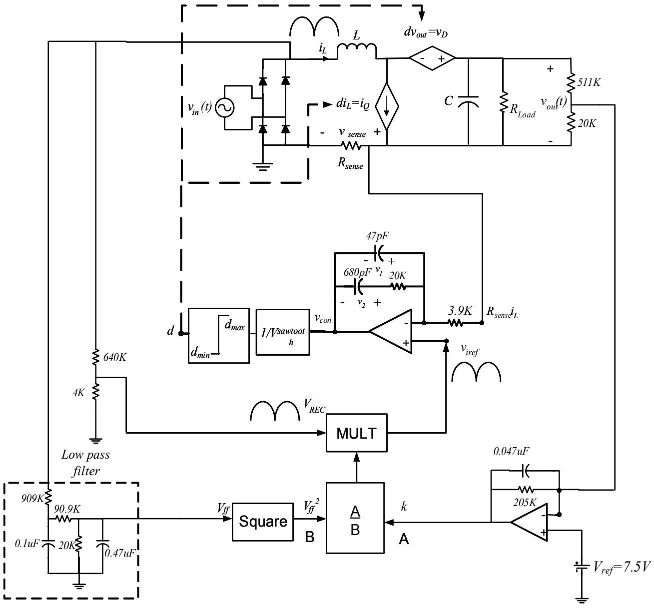


Fig. 6. The proposed single-phase Boost PFC model.

$$v_{iref} = \frac{kV_{rec}}{v_{ff}^{2}} \tag{9}$$

V_{ff} is the feed-forward voltage and k is the output of the error amplifier of the voltage loop. The multiplier multiplies the rectified input voltage with the signal produced by the output of the voltage error amplifier divided by V_{ff}^2. The resulting current control signal v_{iref} has the shape of the input voltage and an average value that controls the output voltage. Readers should take note that the model in Fig. 6 can be modified for converters such as the Buck converter by replacing the averaged model of the Boost with the averaged model of the corresponding converter. The proposed model can be used to predict the steady-state and large-signal dynamic characteristics of the PFC converter.

VI. MODEL VERIFICATION

The simulation and measured large signal waveforms for output voltage and input current were compared between the models and the corresponding prototype. A phase plane plot is useful in analyzing the large signal characteristics of the DC-to-DC converter model [1]. The parameters of the DC-to-DC converter model are that listed in tables 1 and 2 except that the reference current signal is stepped between 0.5A and 1A. From Fig 7, it can be seen that the input current responds quickly when the control signal v_{iref} is stepped from 0.5A to 1A and the output voltage changes quite slowly. Similar response of the input current and output voltage occurs when the control signal v_{iref} is stepped down from 1A to 0.5A. The faster response of the input current due to the change in reference signal v_{iref} is reasonable since the input current i_L is directly controlled by the average current control scheme.

A comparison of the dynamic responses of the simulation using the proposed DC-to-DC converter model and prototype measurements was conducted. Fig. 8 illustrates the transient response when a step change of 0.5A to 1A in the current reference is applied. It is shown in Fig. 8 that the transient waveform obtained from the simulation and experimental results matches quite well with each other. The experimental results presented in Fig. 7 and Fig. 8 is averaged to remove the switching ripple. When v_{iref} is stepped from 0.5A to 1A, the output voltage increases from 22V to 30V and the input current changes from 0.5A to 1A. This shows that the proposed model can successfully predict the response of the Boost converter with average current control, even with a large change in the control signal v_{iref}.

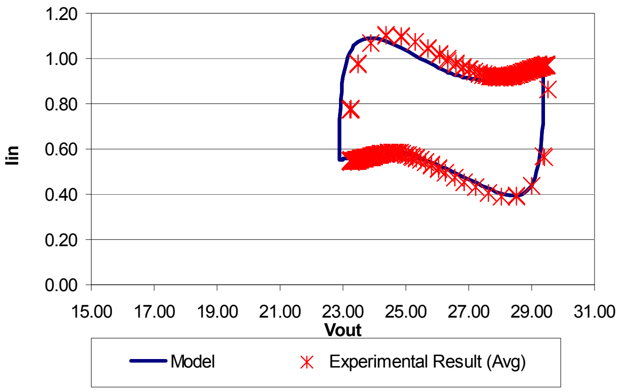
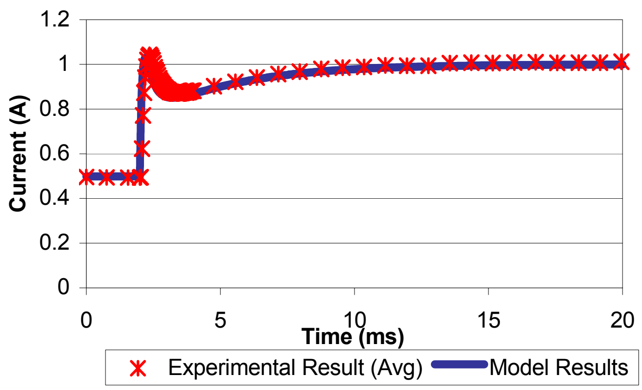
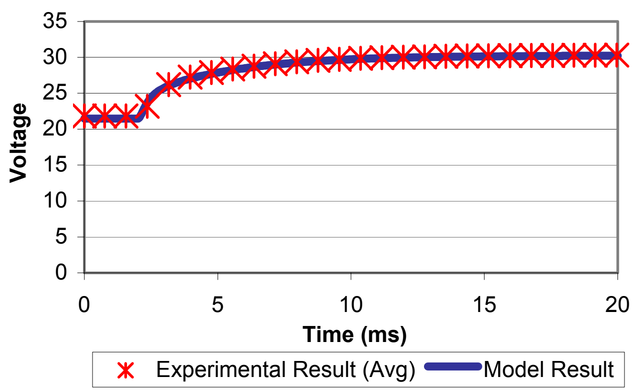


Fig. 7. Phase plane plot when the control signal v_{iref} is stepped between 0.5A and 1A – solid line: model simulation, dashed line: experimental results



(a)



(b)

Fig. 8. Simulation and experimental results when a step change in the reference current signal is applied: (a) Transient input current response when the reference current is stepped from 0.5A to 1A. (b) Transient output voltage response when the reference current is stepped from 0.5A to 1A.

To validate the AC-to-DC PFC model, a Boost PFC circuit using the UC3854 control chip with average current control was constructed. The simulation results from the proposed model were compared to the experimental results (Fig. 9 - Fig. 12). The power train parameters the PFC circuit and model are listed in table 3 and table 4.

Table 1
Parameters of the DC-to-DC Boost Converter

Input Voltage [V_{in}]	Output Voltage	Inductor	Output Capacitor	Switching Frequency	Load Resistor [R_{load}]	Sensing Resistor [R_{sense}]
15V	30V	0.6mH	40µF	100kHz	62Ω	0.27Ω

Table 2
Parameters of the Average Current Control Scheme for the DC-to-DC Boost Converter

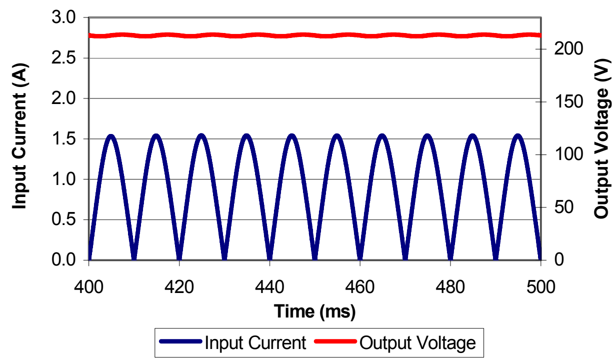
v_{iref}	C_1	C_2	R_1	R_2	$V_{sawtooth}$
1A	82pF	150nF	10kΩ	2.5kΩ	3V

Table 3
Parameters of the PFC Circuit and PFC Model

Input Voltage [V_{in}]	Input Voltage Frequency	Output Voltage	Output Power
120Vrms	50Hz	215V	250W

Table 4
Component Values of the PFC Circuit and PFC Model

Inductor	Output Capacitor	Load Resistor [R_{load}]	Sensing Resistor [R_{sense}]
0.6mH	1120uF	400 Ω	0.25 Ω



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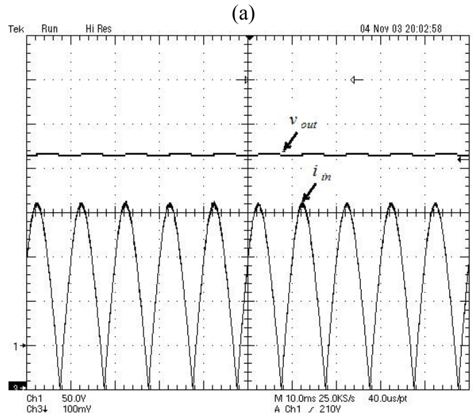
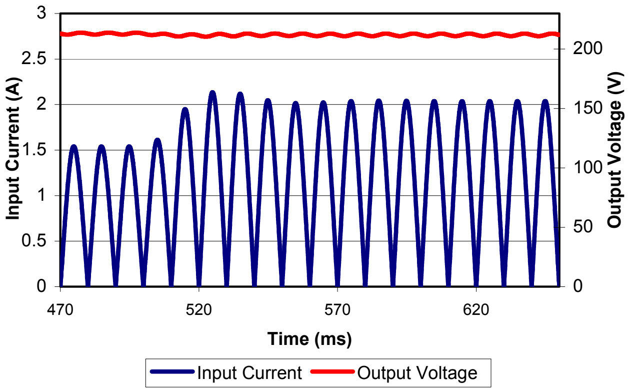


(b)

Fig. 9. The steady-state input current and output voltage waveforms: (a) Model simulation (b) Experimental results

Fig. 9 shows the steady-state waveforms of the inductor current and the output voltage obtained from the model and the prototype. It shows that the model correctly predicts the steady-state output voltage and the inductor current. Fig. 9 illustrates that the proposed model correctly predicts the steady-state output voltage of 215V for the prototype circuit. The experimental inductor current waveform was measured by examining the voltage across the 0.25Ω sensing resister in the prototype PFC circuit. The model also correctly predicts the peak current of around 1.6A which can observed from the experimental results in Fig. 9(b).



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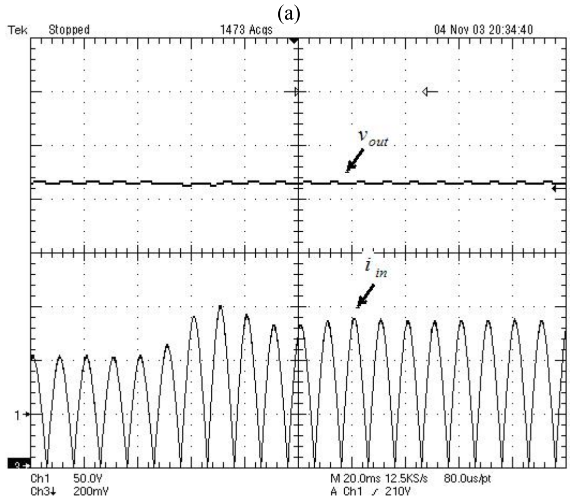
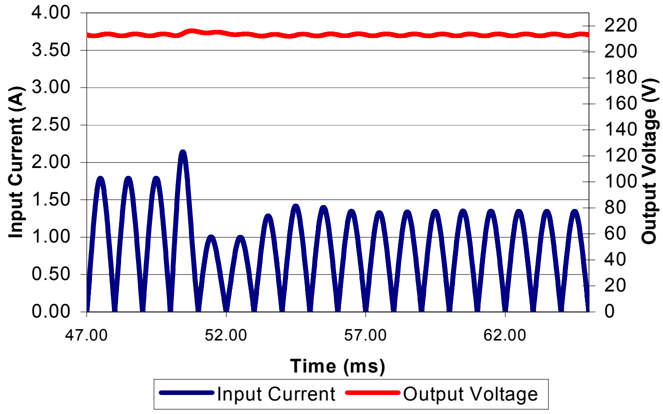


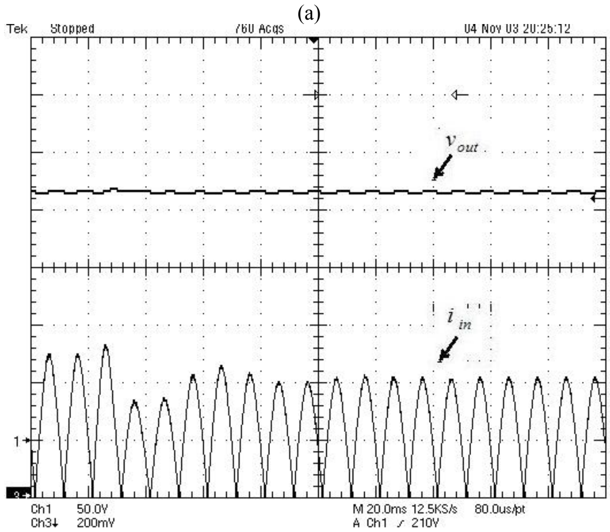
(b)

Fig. 10. Results for I_{in} and V_{out} when R_{Load} is stepped from 400Ω to 300Ω: (a) Model simulation (b) Experimental results

Fig. 10 illustrates the large-signal transient response when the load resistor is stepped from 400Ω to 300Ω. It can be observed that the output voltage dips due to the sudden increase in the amount of current drawn from the output capacitor. However, the voltage loop is able to compensate and maintain the output voltage to the 215V specified by V_{ref} after a brief transient period. The experimental results show that the proposed model can predict the dynamic response of the output voltage and input current of the single-phase PFC circuit when there is a step in the load.



(a)



(b)

Fig. 11. Results for I_{in} and V_{out} when V_{in} is stepped from 95V to 110V RMS:(a) Model simulation (b) Experimental results

Fig. 11 shows the transient response when the input voltage is stepped from 95Vrms to 110Vrms. From Fig. 11, it can be observed that the proposed model correctly predicts the transient output voltage and inductor current response. Both the proposed model and the experimental results show that the magnitude of the inductor current reduces due to the increase in input voltage and the fact that the output power is constant.

Fig. 12 illustrates the large-signal transient response when the output voltage is stepped from 215V to 225V. It can be observed that the input current suddenly increases. This sudden increase in the amount of input current drawn is due to the output voltage loop compensating the signal v_{iref} so that the output voltage reaches the new specified voltage of 225V. The voltage loop is able to compensate change the output voltage to the new specified output voltage of 225V after a brief transient period. The experimental results show that the proposed model can predict the dynamic response of the output voltage and input current when its output voltage is stepped for the AC-to-DC PFC circuit.

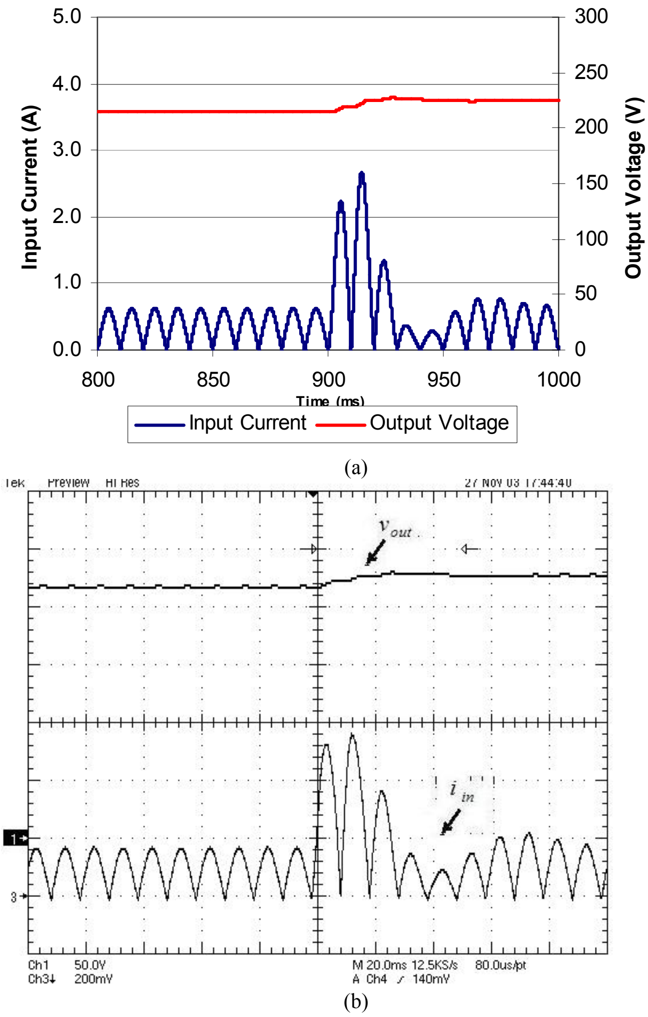


Fig. 12. Results for I_{in} and V_{out} when V_{out} is stepped from 215V to 225V: (a) Model simulation (b) Experimental results

VII. CONCLUSION

In this paper, a large signal model for DC-to-DC and AC-to-DC PFC circuit using average current control has been presented. The proposed model was verified by prototype circuits. For the DC-to-DC converter with average current control model, the large signal behavior was analyzed by both phase plane plot and large signal transient response when a step change is applied to the current reference. The simulated waveforms were compared to the experimental results obtained from the prototype. The measured small, large and steady-state signal responses of the prototype circuit verified the accuracy of the model.

From the experimental results, it can also be observed that the proposed AC-to-DC PFC model can predict the steady-state and transient response of the AC-to-DC PFC circuit. This model is obtained by combining the averaged model of the Boost converter for the power stage and the PFC control scheme using average current control with the PWM circuit model.

The models presented in this paper can accurately predict the system's dynamic response which will aid designers in the control design. Its compatibility with simulation software allows easy analysis of steady-state and large signal characteristics. With this model, accurate predictions of the behavior of the real circuit are obtained and an excellent understanding of the circuit is achieved.

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